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CS/M.Tech(EE)/SEM-1/CI-1.1/2009-10

2009

ADVANCED MATHEMATICAL TECHNIQUES

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) The Cauchy-Reimann equations for the functions $u(x, y)$ and $v(x, y)$ are

a) $u_x = v_y$ and $u_y = -v_x$

b) $u_x = -v_x$ and $u_y = v_y$

c) $u_x = v_y$ and $u_y = v_x$

d) $u_x = -v_y$ and $u_y = v_x$.

ii) The fixed points of the function $f(z) = z/(2 - z)$ are

a) 0 and 2 b) 1 and 2

c) 0 and 1 d) only 2.

iii) If for any two independent events A and B, $P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{3}$, then $P(AB)$ is

a) $\frac{1}{2}$ b) $\frac{3}{2}$

c) $\frac{2}{3}$ d) $\frac{1}{6}$.

iv) The mean and variance of a Binomial $\left(100, \frac{1}{2}\right)$ distribution are

a) 50 and 25 b) 100 and 25

c) 100 and 50 d) 100 and $\frac{1}{2}$.

v) For the shift operator E and forward difference operator Δ,

a) $\Delta = E - 1$ b) $\Delta = E + 1$

c) $\Delta = E$ d) $\Delta^2 = E + 1$.

vi) If $P(A) = P(B) = 1$, then the value of $P(AB)$ is

a) 2 b) $\frac{1}{2}$

c) 1 d) 0.

vii) The distribution for which mean and variance are equal is

a) Poisson b) normal

c) binomial d) exponential.

viii) The value of $\oint_C z^2 \, dz$ where $C : | Z | = 1$ taken anticlockwise is

a) $2\pi i$ b) $-2\pi i$

c) 0 d) π .

ix) The interpolation formula which can be used to find a polynomial from the

given data

x :	0	1	2	4
y :	3	9	17	22

is

a) Newton's forward interpolation formula

b) Gaussian interpolation formula

c) Newton's backward interpolation formula

d) Lagrange's interpolation formula.

x) If two functions $u (x, y)$ and $v (x, y)$ are Harmonic, then $f (z) = u + iv$ is analytic. This statement is

a) True b) False.

xi) If a function $f(z)$ is analytic within and on a contour C, then $\int f(z)\,dz$ is

a) path dependent b) path independent.

xii) The normal curve with parameters m and σ is

a) symmetric about the line $x = 0$

b) symmetric about the line $x = m$

c) not symmetric

d) symmetric about the y-axis.

GROUP – B

Answer any *four* of the following. $4 \times 5 = 20$

2. Derive the mean and variance of Poisson Distribution.

3. Find the analytic function $f(z) = u + iv$, the real part of which is given by $u(x, y) = e^x (x \cos y - y \sin y)$.

4. Find out the polynomial which satisfies the following data :

x :	0	1	2	3
y :	0	1	4	10

5. Evaluate $\oint_C \frac{dz}{(z^2+1)(z^2+4)}$ where C is the contour $|z| = 1{\cdot}5$ taken anti-clockwise.

6. In a shooting competition, the probability of hitting a target of a man is $\frac{1}{5}$. If he fires 5 times, what is the probability of hitting the target at least twice.

7. Prove that (i) $E^{-1} = 1 - \nabla$ (ii) $\Delta E = E\nabla$, where E is the shift operator, Δ is the forward difference operator and ∇ is the backward difference operator.

GROUP – C

Answer any *four* of the following. $4 \times 10 = 40$

8. a) Find the value of cosh 0·56 from the given data :

x_j	$f_j = \cosh x_j$
0·5	1·127626
0·6	1·85465
0·7	1·255169
0·8	1·337435

7

b) Show that $\oint_C \frac{dz}{z-2} = 2\pi i$, where C is the contour $|z - 2| = 2$ taken anti-clockwise. 3

9. a) A box contains 8 red and 5 black balls. Two draws of 3 balls are made successively without replacement. What is the probability that both the drawings give red balls. 4

b) Evaluate $\oint_C \frac{z^3\,dz}{z^3 - 4z^2 + 5z - 2}$, where C is the contour $|z| = 3$ taken anti-clockwise. 6

10. a) Calculate the dominant eigenvalue of the matrix

$$A = \begin{bmatrix} 4 & 4 & 0 \\ 4 & 4 & 0 \\ 0 & 0 & 8 \end{bmatrix}$$ 5

b) A continuous random variable X has probability density function given by

$$f(x) = \begin{cases} kx\,e^{-2x}, & x \geq 0 \\ 0, & \text{otherwise.} \end{cases}$$

Determine the constant k and mean of x. 5

11. a) Determine whether the following functions are Harmonic or not (i) $u = \frac{x}{x^2 + y^2}$ (ii) $v = x^3 - 3x^2$.

If yes, find the analytic function $f(z) = u + iv$. 5

b) Determine the region of the co-plane into which the rectangular region in the z-plane bounded by the lines $x = 0$, $y = 0$, $x = 1$ and $y = 2$ is mapped under the transformation $w = z + (2 - i)$. 5

12. a) Derive the mean of binomial distribution with parameters n and p. 4

b) The heights (in inches) of a group of 10,000 men are normally distributed with parameters $m = 64$ and $\sigma = 4$. Find the number of men whose height is (i) more than $72''$ and (ii) less than $56''$.

[Given : The area under the stdard normal curve to the right of the ordinate at $z = 2$ is ·02.] 6

13. Solve the initial value problem $\frac{dy}{dx} = -0{\cdot}2xy$, $y(0) = 1$ by Runge - Kutta method (fourth order) with $h = 0{\cdot}2$, for 10 steps. Calculate the error in each step. 10