



Name :

Roll No. :

Invigilator's Signature :

**CS/M.Tech(EE)/SEM-1/CI-1.1/2009-10
2009**

ADVANCED MATHEMATICAL TECHNIQUES

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following :
10 × 1 = 10

i) The Cauchy-Reimann equations for the functions

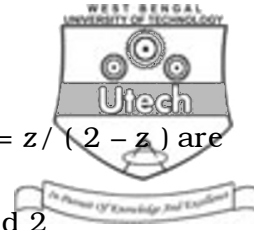
$u(x, y)$ and $v(x, y)$ are

a) $u_x = v_y$ and $u_y = -v_x$

b) $u_x = -v_x$ and $u_y = v_y$

c) $u_x = v_y$ and $u_y = v_x$

d) $u_x = -v_y$ and $u_y = v_x$.



- ii) The fixed points of the function $f(z) = z/(2 - z)$ are
- a) 0 and 2 b) 1 and 2
- c) 0 and 1 d) only 2.
- iii) If for any two independent events A and B , $P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{3}$, then $P(AB)$ is
- a) $\frac{1}{2}$ b) $\frac{3}{2}$
- c) $\frac{2}{3}$ d) $\frac{1}{6}$.
- iv) The mean and variance of a Binomial $\left(100, \frac{1}{2}\right)$ distribution are
- a) 50 and 25 b) 100 and 25
- c) 100 and 50 d) 100 and $\frac{1}{2}$.
- v) For the shift operator E and forward difference operator Δ ,
- a) $\Delta = E - 1$ b) $\Delta = E + 1$
- c) $\Delta = E$ d) $\Delta^2 = E + 1$.
- vi) If $P(A) = P(B) = 1$, then the value of $P(AB)$ is
- a) 2 b) $\frac{1}{2}$
- c) 1 d) 0.

a) Poisson b) normal

c) binomial d) exponential.

a) $2\pi i$
c) 0

b) $-2\pi i$
d) π .

- Newton's forward interpolation formula
- Gaussian interpolation formula
- Newton's backward interpolation formula
- Lagrange's interpolation formula.

a) True b) False.



xi) If a function $f(z)$ is analytic within and on a contour C , then $\int_C f(z) dz$ is

- a) path dependent b) path independent.

xii) The normal curve with parameters m and σ is

- a) symmetric about the line $x = 0$
 b) symmetric about the line $x = m$
 c) not symmetric
 d) symmetric about the y -axis.

GROUP – B

Answer any *four* of the following. $4 \times 5 = 20$

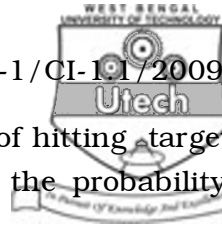
2. Derive the mean and variance of Poisson Distribution.
3. Find the analytic function $f(z) = u + iv$, the real part of which is given by $u(x, y) = e^x (x \cos y - y \sin y)$.
4. Find out the polynomial which satisfies the following data :

$$x : \quad 0 \quad 1 \quad 2 \quad 3$$

$$y : \quad 0 \quad 1 \quad 4 \quad 10$$

5. Evaluate $\oint_C \frac{dz}{(z^2 + 1)(z^2 + 4)}$ where C is the contour

$|z| = 1.5$ taken anti-clockwise.



6. In a shooting competition, the probability of hitting target of a man is $\frac{1}{5}$. If he fires 5 times, what is the probability of hitting the target at least twice.
7. Prove that (i) $E^{-1} = 1 - \Delta$ (ii) $\Delta E = E\Delta$, where E is the shift operator, Δ is the forward difference operator and Δ is the backward difference operator.

GROUP – C

Answer any *four* of the following. $4 \times 10 = 40$

8. a) Find the value of $\cosh 0.56$ from the given data :

$$x_j \quad f_j = \cosh x_j$$

$$0.5 \quad 1.127626$$

$$0.6 \quad 1.85465$$

$$0.7 \quad 1.255169$$

$$0.8 \quad 1.337435$$

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- b) Show that $\oint_C \frac{dz}{z-2} = 2\pi i$, where C is the contour

$$|z-2| = 2 \text{ taken anti-clockwise.} \quad 3$$

9. a) A box contains 8 red and 5 black balls. Two draws of 3 balls are made successively without replacement. What is the probability that both the drawings give red balls. 4



- b) Evaluate $\oint_C \frac{z^3 dz}{z^3 - 4z^2 + 5z - 2}$, where C is the contour $|z| = 3$ taken anti-clockwise. 6

10. a) Calculate the dominant eigenvalue of the matrix

$$A = \begin{bmatrix} 4 & 4 & 0 \\ 4 & 4 & 0 \\ 0 & 0 & 8 \end{bmatrix} \quad 5$$

- b) A continuous random variable X has probability density function given by

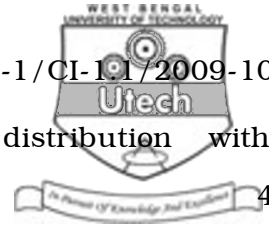
$$f(x) = \begin{cases} kx e^{-2x}, & x \geq 0 \\ 0, & \text{otherwise.} \end{cases}$$

Determine the constant k and mean of x . 5

11. a) Determine whether the following functions are Harmonic or not (i) $u = \frac{x}{x^2 + y^2}$ (ii) $v = x^3 - 3x^2$.

If yes, find the analytic function $f(z) = u + iv$. 5

- b) Determine the region of the z -plane into which the rectangular region in the z -plane bounded by the lines $x = 0$, $y = 0$, $x = 1$ and $y = 2$ is mapped under the transformation $w = z + (2 - i)$. 5



12. a) Derive the mean of binomial distribution with parameters n and p . 4

b) The heights (in inches) of a group of 10,000 men are normally distributed with parameters $m = 64$ and $\sigma = 4$. Find the number of men whose height is (i) more than 72 " and (ii) less than 56 " .

[Given : The area under the stdard normal curve to the right of the ordinate at $z = 2$ is $\cdot 02$.] 6

13. Solve the initial value problem $\frac{dy}{dx} = -0.2xy$, $y(0) = 1$ by Runge - Kutta method (fourth order) with $h = 0.2$, for 10 steps. Calculate the error in each step. 10

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