Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS / M.Tech(EE) / SEM-1 / EMM-101 / 2012-13

2012

ADVANCED ENGINEERING MATHEMATICS

Time Allotted : 3 Hours *Full Marks :* 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Graph sheet(s) will be supplied by Institute on demand.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : 10 × 1 = 10

i) Which of the following relations is not true ?

a) $\frac{\Delta}{\nabla} = \Delta + \nabla$ b) $\frac{\Delta}{\nabla} = \Delta - \nabla$

c) $\nabla = E - 1$ d) $\nabla = 1 - E^{-1}$

ii) Which one is true relations ?

a) $\Delta = 1 - E$ b) $\nabla = 1 - E^{-1}$

c) $\frac{\Delta}{\nabla} = \Delta - \nabla$ d) none of these.

iii) Interpolation in regular interval at the end of the table is done using

a) Newton forward formula

b) Newton backward formula

c) Newton's divided difference formula

d) Gauss forward formula.

iv) For unequal interval which of the following formulae is not used ?

a) Newton's divided difference formula

b) Lagrange interpolation formula

c) NG Forward

d) Bessel interpolation formula.

v) Lagrange interpolation formula is used

a) near the beginning of the table

b) near the end of he table

c) in the middle point of the table

d) at all three.

vi) An LPP having an artificial variable at positive level in the basis, when all $Z_j - C_j \geq 0$ has

a) Unbounded solution

b) Infeasible solution

c) Basic feasible solution

d) Alternate optimal solution

vii) The dual of the dual of an LPP is the

a) Primal b) Dual

c) Both primal and dual d) none of these.

viii) Given that $P(A) = 1/3$, $P(B) = \frac{1}{4}$, $P(A/B) = 1/6$, then following $P(B/A)$ is equal to

a) 1/8 b) 1/7

c) 1/6 d) none of these.

ix) A coin is tossed six times. The probability of obtaining heads and tails alternately is

a) 1/16 b) 1/32

c) 1/8 d) none of these.

x) The p.d.f. of a variable is defined as $f(x) = cx(2-x)$, $0 < x < 2$, then c is

a) 3/4 b) 2/3

c) 1/3 d) none of these.

xi) If $f(z) = \bar{z}$, then $f'(0)$ is

a) 1 b) – 1

c) 0 d) does not exist.

xii) If $f(z) = u(x,y) + iv(x,y)$, then $f'(z)$ is

a) $\dfrac{\partial v}{\partial y} - i\dfrac{\partial u}{\partial y}$ b) $\dfrac{\partial u}{\partial x} - i\dfrac{\partial v}{\partial x}$

c) $\dfrac{\partial u}{\partial y} + i\dfrac{\partial v}{\partial x}$ d) $\dfrac{\partial u}{\partial x} + i\dfrac{\partial v}{\partial y}$.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Solve, by Euler's method, the equation $\dfrac{dy}{dx} = x + y$, $y(0) = 0$, Choose $h = 0{\cdot}2$ and compute $y(0{\cdot}4)$ and y (0(6).

3. a) Find the value of y (3) from the following data :

x	0	2	4	6	8
y	–1	13	43	89	151

b) Interpolate the missing entry in the following data :

x	1981	1982	1983	1984	1985	1986	1987
y	33·1	172·8	219·7	...	375·2	409·6	491·3

4. Find by Lagrange's formula the approximate from of the function $y = f(x)$, given by

x	0	2	3	6
$f(x)$	659	705	729	804

5. Find the value of the constant k such that

$$f(x) = kx(1-x), \quad 0 < x < 1$$

$$= 0, \qquad \textit{elsewhere}$$

is a possible density function and compute $P(x > \frac{1}{2})$. Also find $E(x)$.

6. If $u - v = (x - y)(x^2 + 4xy + y^2)$ and $f(z) = u + iv$ is an analytic function of $z = x + iy$, find $f(z)$ in terms of z.

7. Solve the following L.P.P.

Minimize $Z = 4x + 8y + 3z$

s.t. $x + y \geq 2$

$2x + z \geq 5$

$x, y, z \geq 0.$

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

8. a) Evaluate $\frac{1}{2\pi i} \int_C \frac{e^{zt}}{(z^2+1)} dz$, if $t > 0$ and c is the circle : $| z | = 3$

b) Using Cauchy's Residue theorem, prove that

$$\int_0^{2\pi} \frac{\sin^2 \theta}{a + b\cos\theta} d\theta = \frac{2\pi}{b^2}\left[a - \sqrt{a^2 - b^2}\right];\ a > 0,\ b > 0$$

9. a) Use Runge-Kutta method of the fourth order to find $y(0{\cdot}2)$.

Given that $y\frac{dy}{dx} = y^2 - x$, $y(0) = 2$, by taking $h = 0{\cdot}2$.

b) A company manufactures two products A and B. Each unit of B takes twice as long to produce as one unit of A and if the company was to produce only A, it would have time to produce 2,000 units per day. The availability of the raw material is sufficient to produce 1,500 units per day of both A and B combined. Product B requiring a special ingredient, only 600 units can be made per day. If A fetches a profit of Rs. 2 per unit and B a profit of Rs. 4 per unit, find the optimum product max by graphical method.

10. a) Solve the transportation problem and checking the optimality, find the optimal solution :

D1	D2	D3
4	3	2
1	5	0
3	8	5

b) Solve the assignment problem :

D1	D2	D3	D4	D5	D6
A	6	5	8	11	16
B	1	13	16	1	10
C	16	11	8	8	8
D	9	14	12	10	16
E	10	13	11	8	16

11. a) Find the correlation coefficient between the two variables x and y where :

X :	1	2	3	4	5	6	7	8
Y :	74	54	52	51	52	53	58	71

b) There are two identical urns containing respectively 4 white, 3 red balls and 3 white, 7 red balls. An urn is chosen at random and a ball is drawn from it. Find the probability that the ball is white. If the ball drawn is white, what is the probability that it is from the first urn ?

12. a) Compute by the Newton-Raphson method the positive root of the equation $3x^2 + 2x = 9$, correct up to four significant figures.

b) Evaluate $\oint_C \frac{e^z}{(z+1)(z+2)} dz$ where C is the circle : $| z - 1 | = 4$