



Name : .....

Roll No. : .....

Invigilator's Signature : .....

**CS/M.Tech(EE)/SEM-1/EMM-101/2010-11  
2010-11**

**ADVANCED ENGINEERING MATHEMATICS**

Time Allotted : 3 Hours

Full Marks : 70

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words  
as far as practicable.*

1. Answer any seven of the following :  $7 \times 2 = 14$

i) Find the rank of the matrix  $A = \begin{pmatrix} 1 & 0 & 3 \\ 4 & -1 & 5 \\ 2 & 0 & 6 \end{pmatrix}$ .

ii) Find the value of  $\lambda$  if the matrix

$$A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & \lambda \end{pmatrix} \text{ is singular.}$$

iii) Find the value of  $A^{100}$  if  $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ .

iv) State whether the following functions are analytic or not :

a)  $f(z) = \operatorname{Re}(z)$

b)  $f(z) = z^2$ .

v) Evaluate :  $\oint_C \frac{e^{2z} dz}{(z-1)(z-2)}$ , where  $C$  is the circle

$$|z| = 3.$$



- vi) Solve the following system of equations by Gauss elimination method :

$$5x_1 - x_2 + x_3 = 10$$

$$2x_1 + 4x_2 = 12$$

$$x_1 + x_2 + 5x_3 = -1$$

- vii) Prove that

a)  $E = \frac{1}{1 - \square}$

b)  $D = \frac{1}{h} \log E,$

where  $E$  is the shift operator,  $\square$  is the backward difference operator and  $D$  is the differential operator,  $h$  being the shift in  $x$ .

- viii) Evaluate  $\oint_L \operatorname{Re}(z) dz$  where  $L$  is the line joining the origin to the point  $(1 + i)$ .

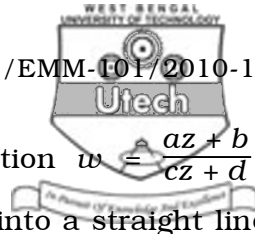
- ix) Classify the stationary points of the function :

$$f(x, y) = 2x^2 + 2xy + y^2 - 2x - 2y + 5.$$

- x) Find the residue of  $f(z) = \frac{4 - 3z}{z^2 - z}$  at the poles  $z = 0$  and  $z = 1$ .

Answer any *eight* of the following :  $8 \times 7 = 56$

2. a) If  $u = x^3 - 3xy^2$ , then show that there exists a function  $v(x, y)$  such that  $w = u + iv$  is analytic in a finite region.
- b) Find the bilinear transformation which maps the points  $z = \bullet, i, o$  into the points  $w = o, i, \bullet$  respectively.



3. Find the condition that the transformation  $w = \frac{az + b}{cz + d}$  transforms the unit circle in the  $w$ -plane into a straight line in the  $z$ -plane.
4. a) Find the poles of the function  $f(z) = \frac{1}{\sin z - \cos z}$ . Also specify the nature of the poles.
- b) Find the zeros of the following  $f(z) = z^2 \sin 2z$  and indicate its nature.
5. a) Evaluate :  $\oint_C \frac{z \, dz}{(z-1)(z-2)^2}$ , where  $C$  is  $|z-2| = \frac{1}{2}$  taken anti clockwise.
- b) Evaluate :  $\oint_C \frac{z+1}{z^2-2z} \, dz$ , where  $C$  is the circle  $|z| = 5$ .
6. Find the eigen values and eigen vectors of the matrix
- $$A = \begin{pmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{pmatrix}.$$
7. a) If  $H = P + iQ$  be a Hermitian matrix, then show that  $P$  is a real symmetric matrix and  $Q$  is a real skew-symmetric matrix.
- b) If  $S = M + iN$  be a skew-Hermitian matrix, then show that  $M$  is a real skew-symmetric matrix and  $N$  is a real symmetric matrix.



8. Show that if the matrices  $A$  and  $B$  are orthogonal and  $|A| + |B| = 0$  then  $A + B$  is singular.
9. Apply the Newton - Raphson method to find a root of the equation  $x^2 - 5x + 4 = 0$  with trial value 5 correct up to 3 places of decimal.
10. a) Find  $e^{-0.75}$  from the following data using both Newton's forward and backward formulae :

$x :$	1.00	1.25	1.50	1.75	2.00
$e^{-x} = y :$	0.3679	0.2865	0.2231	0.1738	0.1353

- b) Use Runge-Kutta method of 4th order to find  $y ( 0.2 )$  and compare it with the exact solution of  $y \frac{dy}{dx} = y^2 - x ; y ( 0 ) = 2$  taking  $h = 0.2$ .
11. Find the extreme values of  $f ( x, y ) = 1 - x^2 - y^2$  subject to the condition  $x + y = 1$ .
12. Prove that the shortest distance between two points in a plane is a straight line.

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