Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/M.Tech(EE)/SEM-1/EMM-101/2010-11

2010-11

ADVANCED ENGINEERING MATHEMATICS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer any *seven* of the following : $7 \times 2 = 14$

   i) Find the rank of the matrix $A = \begin{pmatrix} 1 & 0 & 3 \\ 4 & -1 & 5 \\ 2 & 0 & 6 \end{pmatrix}$.

   ii) Find the value of λ if the matrix

   $$A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & \lambda \end{pmatrix} \text{ is singular.}$$

   iii) Find the value of A^{100} if $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$.

   iv) State whether the following functions are analytic or not :

   a) $f(z) = Re(z)$

   b) $f(z) = z^2$.

   v) Evaluate : $\oint_C \frac{e^{2z}\,dz}{(z-1)(z-2)}$, where C is the circle $|z| = 3$.

vi) Solve the following system of equations by Gauss elimination method :

$$5x_1 - x_2 + x_3 = 10$$

$$2x_1 + 4x_2 = 12$$

$$x_1 + x_2 + 5x_3 = -1$$

vii) Prove that

a) $E = \dfrac{1}{1 - \nabla}$

b) $D = \dfrac{1}{h} \log E$,

where E is the shift operator, ∇ is the backward difference operator and D is the differential opeator, h being the shift in x.

viii) Evaluate $\oint_L Re\,(z)\,dz$ where L is the line joining the origin to the point $(1 + i)$.

ix) Classify the stationary points of the function :

$$f(x, y) = 2x^2 + 2xy + y^2 - 2x - 2y + 5.$$

x) Find the residue of $f(z) = \dfrac{4 - 3z}{z^2 - z}$ at the poles $z = 0$ and $z = 1$.

Answer any *eight* of the following : $8 \times 7 = 56$

2. a) If $u = x^3 - 3xy^2$, then show that there exists a function $v(x, y)$ such that $w = u + iv$ is analytic in a finite region.

b) Find the bilinear transformation which maps the points $z = \infty, i, o$ into the points $w = o, i, \infty$ respectively.

3. Find the condition that the transformation $w = \dfrac{az+b}{cz+d}$ transforms the unit circle in the w-plane into a straight line in the z-plane.

4. a) Find the poles of the function $f(z) = \dfrac{1}{\sin z - \cos z}$. Also specify the nature of the poles.

   b) Find the zeros of the following $f(z) = z^2 \sin 2z$ and indicate its nature.

5. a) Evaluate : $\oint_C \dfrac{z\,dz}{(z-1)(z-2)^2}$, where C is $|z-2| = \frac{1}{2}$ taken anti clockwise.

   b) Evaluate : $\oint_C \dfrac{z+1}{z^2-2z}\,dz$, where C is the circle $|z| = 5$.

6. Find the eigen values and eigen vectors of the matrix

$$A = \begin{pmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{pmatrix}.$$

7. a) If $H = P + iQ$ be a Hermitian matrix, then show that P is a real symmetric matrix and Q is a real skew-symmetric matrix.

   b) If $S = M + i\,N$ be a skew-Hermitian matrix, then show that M is a real skew-symetric matrix and N is a real symmetric matrix.

8. Show that if the matrices A and B are orthogonal and $|A| + |B| = 0$ then $A + B$ is singular.

9. Apply the Newton - Raphson method to find a root of the equation $x^2 - 5x + 4 = 0$ with trial value 5 correct up to 3 places of decimal.

10. a) Find $e^{-0.75}$ from the following data using both Newton's forward and backward formulae :

x :	1·00	1·25	1·50	1·75	2·00
$e^{-x} = y$:	0·3679	0·2865	0·2231	0·1738	0·1353

b) Use Runge-Kutta method of 4th order to find $y(0.2)$ and compare it with the exact solution of $y\frac{dy}{dx} = y^2 - x$; $y(0) = 2$ taking $h = 0.2$.

11. Find the extreme values of $f(x, y) = 1 - x^2 - y^2$ subject to the condition $x + y = 1$.

12. Prove that the shortest distance between two points in a plane is a straight line.