

Name : .....

Roll No. : .....

Invigilator's Signature : .....

**CS/M.TECH (EE)/SEM-1/EAM-101/2012-13  
2012**

**ADVANCED ENGINEERING MATHEMATICS**

Time Allotted : 3 Hours

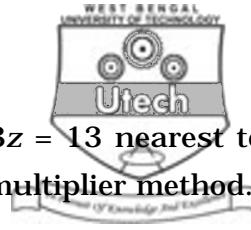
Full Marks : 70

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words  
as far as practicable.*

Answer Question No. 1 and any *four* from the rest.

1. a) Show that  $\lambda^m$  is an eigenvalue of  $A_{n \times n}$  where  $\lambda$  is the eigenvalue of  $A_{n \times n}$ ;  $n \in N$ . 3
- b) State Cauchy's Integral formula. 2
- c) Evaluate  $\oint_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$  where  $C : |z| = 3$ . 3
- d) Define Saddle points. 2
- e) How Runge-Kutta method is related with Euler's method, modified Euler's method and Runge's method in numerical analysis ? 2
- f) Why DFPP is not preferred with respect to SFPP in Laplace's equation ? 2



2. a) Find the point in the plane  $x + 2y + 3z = 13$  nearest to the point  $(1, 1, 1)$  using Lagrange's multiplier method.

7

- b) If  $u = e^{-x} (x \sin y - y \cos y)$ , show that there is another function  $v(x, y)$  such that  $u + iv$  is analytic.

7

3. a) Show that the necessary condition for

$$\int_{x_1}^{x_2} f(x, y, y') dx \text{ to be an extremum is}$$

$$\frac{\partial f}{\partial y} - \frac{d}{dx} \left( \frac{\partial f}{\partial y'} \right) = 0.$$

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- b) Find the value of the integral  $\int_0^{1+i} (x - y + ix^2) dz$

i) along the straight line from  $z = 0$  to  $z = 1 + i$

ii) along the real axis from  $z = 0$  to  $z = 1$  and then along a line parallel to the imaginary axis from  $z = 1$  to  $z = 1 + i$ .

3 + 4

4. a) Find whether the set of vectors

$$S = \{ (1, 2, -1, 3), (3, -1, 2, 1), (2, -2, 3, 2), (1, -1, 1, -1) \}$$

is linearly dependent. Find a subset  $S$  which is linearly independent.

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- b) Show that  $S = \{ (x, y, z) : x^2 + y^2 = z^2 \}$  is not a subspace of  $\mathbb{R}_3(\mathbb{R})$ .

6



5. a) A particle moves on a smooth curve, joining the two fixed points  $A$  and  $B$  under gravity, starting from rest from  $A$ . Find the form of the path in order that the time from  $A$  to  $B$  is minimum. 8
- b) State and prove Cauchy's original theorem. 6
6. a) Use Runge-Kutta method to find  $y$  when  $x = 1.2$  in steps of  $0.1$  given that  $\frac{dy}{dx} = x^2 + y^2$  and  $y(1) = 1.5$ . 8
- b) Obtain Picard's second approximate solution of the initial value problem  $\frac{dy}{dx} = \frac{x^2}{y^2 + 1}$ ,  $y(0) = 0$ . 6
7. a) If  $f(z)$  and  $g(z)$  are analytic within and on a closed curve  $C$ , and  $|g(z)| < |f(z)|$  on  $C$ , then show that  $f(z) + g(z)$  have same number of zeroes inside  $C$ . 9
- b) Find smallest positive root of the equation  $e^x = 4 \sin x$  correct up to 4 decimal places by bisection method. 5
8. Suppose  $\lambda$  be an eigenvalue of an  $n \times n$  square matrix  $A$ . Then show the following :
- i)  $\lambda$  is an eigenvalue of  $A^T$ .
- ii)  $c\lambda$  is also an eigenvalue of  $cA$  for any scalar  $c$ .
- iii)  $\lambda^n$  is an eigenvalue of  $A^n$ .
- iv)  $\lambda^{-1}$  is an eigenvalue of  $A^{-1}$ . 4 + 2 + 4 + 4