Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/M.TECH (EE)/SEM-1/EAM-101/2010-11

2010-11

ADVANCED ENGINEERING MATHEMATICS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer Question No. 1 and any *four* from the rest questions.

1. i) Show that $\log z$ is analytic anywhere except at (0, 0). 2

ii) Convert $\int_0^{2\pi} \frac{d\phi}{a^2 + \sin^2 \phi}$ into an equivalent complex integral. 2

iii) Show that λ^m is *e-val* of A_{nxn} where λ is the eigenvalue of A_{nxn} ; $m \in N$. 3

iv) Find co-ordinates of (0, 3, 1) with respect to the vectors (1, 1, 0), (1, 0, 1), (0, 1, 1). 2

v) Find critical points of $w = e^{2z} - 2iz + 3$. 2

vi) Explain *N-R* method geometrically. 3

2. a) State and prove Morera's Theorem. 8

b) Evaluate $\int_c \frac{e^{2z}}{(z+1)^3}$ dz where c is the ellipse $x^2 + y^2 = 9$. 6

3. a) If $f'(z)$ has a n-fold zero at $z = z_0$ then show that the angles after transformation are multiplied by $(n+1)$. 8

b) Show that Laplace's Equation is preserved under conformal transformation. 6

4. a) Obtain the equation to the curve $y = f(x)$ giving the minimum surface of revolution about X-axis. 7

b) Show that Euler-Lagrange equation can be put in the form :

$$1/y\,[\,d/dx\,(F - \grave{y}\,\partial F / \partial \grave{y}) - \partial F / \partial x\,] = 0. \quad 7$$

5. a) State and prove Caley Hamilton theorem and verify it for the matrix : 8

$\{0\ 0\ 1\ ;\ 3\ 1\ 0\ ;\ -2\ 1\ 4\}$.

b) Show that the eigenvalues of a symmetric matrix are all real. 6

6. a) A top open rectangular box has capacity 32 cubic ft. Find the dimension of the box so that the total surface is minimum. 7

b) Find all extreme values of the function :

$x^3 + y^3 - 3x - 12y + 20$.

Find also the saddle points. 7

7. a) Find whether the set and vectors.

$S = \{ (1, 2, -1, 3), (3, -1, 2, 1), (2, -2, 3, 2), (1, -1, 1, -1) \}$

is linearly dependent. Find a subset of S which is linearly independent. 8

b) Show that $S = \{ (x, y, z) : x^2 + y^2 = z^2 \}$ is not a sub-space of $\mathbb{R}_3 (\mathbb{R})$. 6

8. a) Solve the equation using Gauss-Seidel method : 9

$$28x + 4y - z = 32$$

$$x + 3y + 10z = 24$$

$$2x + 17y + 4z = 35.$$

b) Find the smallest positive root of the equation : 5

$$e^x = 4 \sin x$$

Correct upto 4 decimal places by bisection method.