Name :	Utech
Roll No.:	
Invigilator's Signature :	

CS / M. TECH (EDPS) / SEM-1 / EAM-101 / 2010-11 2010-11

ADVANCED EINGINEERING MATHEMATICS

Time Allotted: 3 Hours Full Marks: 70

The figures in the margin indicate full marks.

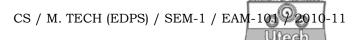
Candidates are required to give their answers in their own words as far as practicable.

GROUP - A

Answer any *two* of the following : $2 \times 14 = 28$

- 1. a) Find the bilinear transformation which maps $z=1,i,-1 \ \text{respectively on to} \ w=i,0,-1. \ \text{Also find}$ the image of |z|=1.
 - b) Evaluate $\int_C \frac{3z^2 + z + 1}{(z^2 1)(z 3)} dz$, C: | z | = 2. 6 + 8
- 2. a) Find a real root of the equation $x^3 3x + 1 = 0$ between 1 and 2 correct to three places of decimal by using bisection method.

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b) Solve the following system of equations using Gauss elimination method:

$$x + y + z = 9$$
$$2x - 3y + 4z = 13$$
$$3x + 4y + 5z = 40$$

8 + 6

3. a) Find a basis and the dimension of the vector space $M = \left\{ \begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix} : x_1 + x_2 + x_3 + x_4 = 0, \ x_i \in R \right\} \quad \text{w.r.t.}$ usual matrix addition and multiplication of a matrix

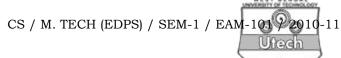
usual matrix addition and multiplication of a matrix by a real scalar.

b) Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{bmatrix}.$$
 6 + 8

- 4. a) Examine the consistency of the following system of equations and solve, if consistent : $x_1 + x_2 + x_3 = 6$, $x_1 x_2 x_3 = -4$, $x_1 + x_2 x_3 = 0$, $2x_1 2x_3 = 4$.
 - b) Determine the linear mapping $T: \mathbb{R}^3 \to \mathbb{R}^3$ that maps the basis vectors (1, 0, 0), (0, 1, 0), (0, 0, 1) of \mathbb{R}^3 to the vectors (0, 1, 0), (0, 0, 1), (1, 0, 0) respectively. Find Ker(T) and Im(T). Verify that dim Ker(T) + dim Im(T) = 3.

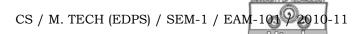
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GROUP - B

Answer any *three* of the following. $3 \times 14 =$

- 5. a) Solve $y_{n+2} 4y_{n+1} + 4y_n = n^2 \cdot 2^n$.
 - b) Using calculus of variation find the curve on which the functional $\int_{0}^{\frac{\pi}{8}} (y'^2 + 2yy' 16y^2) \, dx \quad \text{can} \quad \text{be}$ extremized, where $y' = \frac{dy}{dx}$.
- 6. Apply fourth order Runge-Kutta method to find y (0.01), given that $\frac{dy}{dx} = \frac{2}{x+y}$, y (0) = .5
- 7. Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 4x^2y^2$ over the square region bounded by the lines x = 0 = y, x = 3 = y with u = 0 on the boundary with mesh length 1.
- 8. Find the numerically largest eigenvalue of the matrix $\begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$ by power method.



9. Solve by Lagrangian method:

Optimize

$$f(x_1, x_2, x_3) = 2x_1^2 - 24x_1 + 2x_2^2 - 8x_2 + 2x_3^2 - 12x_3 + 20$$

subject to the constraints $x_1 + x_2 + x_3 = 11$, $x_1, x_2, x_3 \ge 0$.

OR

Solve by Lagrangian method:

Optimize $z = 4x_1 + 9x_2 - x_1^2 - x_2^2$ subject to the constraints

$$4x_1 + 3x_2 = 15$$
, $3x_1 + 5x_2 = 14$, $x_1, x_2 \ge 0$.

10. Solve by gradient method:

Maximize $f(x_1, x_2) = 2x_1x_2 + x_2 - x_1^2 - 2x_2$, starting with (0,0).

14

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