Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS / M. TECH (EDPS) / SEM-1 / EAM-101 / 2010-11

2010-11

ADVANCED EINGINEERING MATHEMATICS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

Answer any *two* of the following : $2 \times 14 = 28$

1. a) Find the bilinear transformation which maps $z = 1, i, -1$ respectively on to $w = i, 0, -1$. Also find the image of $|z| = 1$.

   b) Evaluate $\int_C \frac{3z^2 + z + 1}{(z^2 - 1)(z - 3)} dz$, $C : |z| = 2$. 6 + 8

2. a) Find a real root of the equation $x^3 - 3x + 1 = 0$ between 1 and 2 correct to three places of decimal by using bisection method.

b) Solve the following system of equations using Gauss elimination method :

$$x + y + z = 9$$
$$2x - 3y + 4z = 13$$
$$3x + 4y + 5z = 40$$

8 + 6

3. a) Find a basis and the dimension of the vector space $M = \left\{ \begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix} : x_1 + x_2 + x_3 + x_4 = 0, \ x_i \in R \right\}$ w.r.t. usual matrix addition and multiplication of a matrix by a real scalar.

b) Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{bmatrix}$. 6 + 8

4. a) Examine the consistency of the following system of equations and solve, if consistent : $x_1 + x_2 + x_3 = 6$, $x_1 - x_2 - x_3 = -4$, $x_1 + x_2 - x_3 = 0$, $2x_1 - 2x_3 = 4$.

b) Determine the linear mapping $T : R^3 \to R^3$ that maps the basis vectors (1, 0, 0), (0, 1, 0), (0, 0, 1) of R^3 to the vectors (0, 1, 0), (0, 0, 1), (1, 0, 0) respectively. Find *Ker* (*T*) and *Im* (*T*). Verify that *dim Ker* (*T*) + *dim Im* (*T*) = 3. 6 + 8

GROUP – B

Answer any *three* of the following. $3 \times 14 = 42$

5. a) Solve $y_{n+2} - 4y_{n+1} + 4y_n = n^2 \cdot 2^n$.

   b) Using calculus of variation find the curve on which the functional $\int_0^{\pi/8} (y'^2 + 2yy' - 16y^2)\, dx$ can be extremized, where $y' = \frac{dy}{dx}$. 7 + 7

6. Apply fourth order Runge-Kutta method to find $y(0 \cdot 01)$, given that $\frac{dy}{dx} = \frac{2}{x+y}$, $y(0) = \cdot 5$ 14

7. Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 4x^2y^2$ over the square region bounded by the lines $x = 0 = y$, $x = 3 = y$ with $u = 0$ on the boundary with mesh length 1. 14

8. Find the numerically largest eigenvalue of the matrix $\begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$ by power method. 14

9. Solve by Lagrangian method :

Optimize

$f(x_1, x_2, x_3) = 2x_1^2 - 24x_1 + 2x_2^2 - 8x_2 + 2x_3^2 - 12x_3 + 20$

subject to the constraints $x_1 + x_2 + x_3 = 11$, $x_1, x_2, x_3 \geq 0$.

OR

Solve by Lagrangian method :

Optimize $z = 4x_1 + 9x_2 - x_1^2 - x_2^2$ subject to the constraints $4x_1 + 3x_2 = 15$, $3x_1 + 5x_2 = 14$, $x_1, x_2 \geq 0$. 14

10. Solve by gradient method :

Maximize $f(x_1, x_2) = 2x_1x_2 + x_2 - x_1^2 - 2x_2$, starting with (0,0).

14

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