Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS/M.Tech(ECE)/SEM-2/MCE-202/2013

2013
ERROR CONTROL CODING

Time Allotted : 3 Hours *Full Marks :* 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Note : (i) Missing data may be suitable assumed.

(ii) Illustrate your answers with neat sketches wherever necessary

Answer Question No. 1 and any *four* from the rest.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *seven* of the following : $7 \times 2 = 14$

   i) An analogue channel perturbed by AWGN has a bandwidth B = 25 kHz and a power signal-to-noise ratio SNR of 18 dB. What is the capacity of this channel in bits per second ?

   a) 150053 bps b) 29902 bps

   c) 150000 bps d) 450000 bps.

ii) A (7, 4) Linear Block Code with minimum distance guarantees error detection of

a) ≤ 4 bits b) ≤ 2 bits

c) ≤ 3 bits d) none of these.

iii) If m = 4, then what will be the length of the BCH code ?

a) 16 b) 17

c) 15 d) none of these.

iv) The binary Hamming Codes have the property that

a) $(n, k) = (2^m + 1, 2^m - 1 - m)$

b $(n, k) = (2^m - 1, 2^m - 1 + m)$

c) $(n, k) = (2^m - 1, 2^m - 1 - m)$

d) $(n, k) = (2^m - 1, 2^{m-1} - m)$.

v) For $GF(2^3)$ the elements in the set are

a) { 1, 2, 3, 4, 5, 6, 7 }

b) { 0, 1, 2, 3, 4, 5, 6 }

c) { 0, 1, 2, 3 }

d) { 0, 1, 2, 3, 4, 5, 6, 7 }.

vi) A code with minimum distance $d_{min} = 5$. How many errors it can correct ?

a) 3 b) 4

c) 2 d) 1.

vii) The Hamming distance between v = 1001011 and w = 0100010 is

a) 3 b) 4

c) 2 d) 1.

viii) Consider the parity check matrix $H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$ and the received vector r = (001110).

Then the syndrome is given by

a) (110) b) (100)

c) (111) d) (101).

ix) A (8, 4) linear code has a code rate of

a) 8 b) 4

c) 0·5 d) 2.

GROUP – B

(Long Answer Type Questions)

Answer any *four* of the following. 4 × 14 = 56

2. a) What are irreducible polynomials ? What are their uses ? List the irreducible polynomials of $GF\ (2^3)$ and $GF\ (2^4)$ fields. 9

b) Construct the extension field $GF\ (2^3)$ from the base FG (2) using primitive polynomial $x^3 + x + 1$. Also tabulate the seven non-zero elements. 5

3. a) A binary block code has code vectors in systematic form as given in Table.

Table : A block code table

0	0	0	0	0	0
0	1	1	1	0	0
1	0	1	0	1	0
1	1	0	1	1	0
1	1	0	0	0	1
1	0	1	1	0	1
0	1	1	0	1	1
0	0	0	1	1	1

(i) What is the rate of the code ?

(ii) Write down the generator and parity check matrices of this code in systematic form.

(iii) What is the minimum Hamming distance of the code ?

(iv) How many errors can it correct, and how many can it detect ?

(v) Compute the syndrome vector for the received vector r = (101011) and hence find the location of any error. 8

b) The generator matrix of a binary linear block code is given below :

$$G = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

(i) Write down the parity check equations of the code.

(ii) Determine the code rate and minimum Hamming distance.

(iii) If the error rate at the input of the decoder is 10–3, estimate the error rate at the output of the decoder. 6

4. a) What is standard array ? Explain how the standard array can be used to make a correct decoding. 5

b) The Hamming block code $C_b(15, 11)$ has the following parity check submatrix :

$$P = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

(i) Construct the parity check matrix of the code.

(ii) Construct the error pattern syndrome table.

(iii) Apply syndrome decoding to the received vector $r = (01111001011011)$. 9

5. a) What is meant by a cyclic error-control code ? 5

b) A binary linear cyclic block code $C_{cyc}(n, k)$ has code length $n = 14$ and generator polynomial $g(X) = 1 + X^3 + X^4 + X^5$.

(i) If all the information symbols are '1's, what is the corresponding code vector ?

(ii) Find the syndrome corresponding to an error in the last information symbol. Is this code capable of correcting this error ?

(iii) Can cyclic codes be non-linear ? 9

6. a) Determine the generator polynomial of the binary BCH code $C_{BCH}(31, 16)$ able to correct error patterns of size $t = 3$ or less. 4

b) (i) Show that the shortest binary cyclic BCH code with the generator polynomial $g(X) = (1 + X + X^4)(1 + X + X^2 + X^3 + X^4)$ has code length $n = 15$ and minimum Hamming distance $d_{\min} = 5$.

(ii) Describe the Meggitt or an algebraic decoding method for the above code.

(iii) Use the decoding method you have described to show how errors in the first two positions of a received vector would be corrected. 10

7. a) Use the BCH bound to show that the minimum Hamming distance of the cyclic code with block length $n = 7$ and $g(X) = (X + 1)(1 + X + X^3)$ is 4. What is the minimum Hamming distance if $n = 14$ and why ? 7

b) Show that for a double-error-correcting cyclic code, $s_3 = s_1^2 + s_1^2 \beta l + s_1 \beta l^2$ where $\beta l = \alpha jl$ and hence find the errors in the received vector r = (000100111111011), given that the transmitted vector is from the cyclic BCH code $C_{BCH}(15, 7)$ generated by

$g(X) = (1 + X + X^4)(1 + X + X^2 + X^3 + X^4)$. 7

8. An extended RS code over $GF\ (2^2)$ has the following generator matrix :

$$G = \begin{bmatrix} 1 & \alpha & 1 & 0 & 0 \\ 1 & \alpha^2 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

a) What is the rate of the code and its minimum Hamming distance ?

b) Is $r = (\ \alpha^2 \quad \alpha \quad \alpha \quad 0 \quad 1\ )$ a code vector of this code ?

c) The received vector $r = (\ 0 \quad \alpha \quad 1 \quad \alpha^2 \quad 0\ )$ contains a single error : find its position and value.

9. Write short notes of the following :

a) BCH code

b) Fano decoding

c) Golay code

d) Reed-Solomon code.