



Name :

Roll No. :

Invigilator's Signature :

**CS/M.Tech(ECE)/SEM-1/MCE-101/2009-10
2009**

ENGINEERING MATHEMATICS & STATISTICS

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

Answer Question No. 1 is compulsorily and any *four* of the rest.

1. Answer the following questions with proper justifications :

7 × 2

- a) If $w = f(z) = u(x, y) + i v(x, y)$ be an analytic function in some region of the Z-plane, then show that

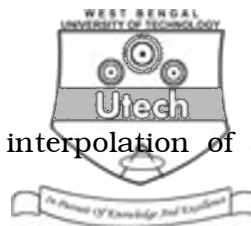
$$\nabla^2 u = \nabla^2 v = 0.$$

- b) Define a pole of order 'n' at the point Z_0 of the function $f(z)$. What is $\lim_{z \rightarrow Z_0} (z - Z_0)^n f(z)$?

- c) How does the difference equation

$$u_{x+2} - (a + b) u_{x+1} + ab u_x = 0$$

arise from the relation $U_x = Aa^x + Bb^x$, A and B being two arbitrary constants ?



- d) Write Newton's formula for forward interpolation of a function of x .
- e) Explain the concept of Newton-Raphson method to determine the approximate values of the real roots of an equation $f(x) = 0$.
- f) Describe the Euler-Lagrange equation for extremisation of the functional integration :

$$\int_{x_0}^{x_1} F\left(x, y, \frac{dy}{dx}\right) dx,$$

stating the necessary conditions to be satisfied.

- g) Describe the classical definition of probability and discuss its limitations.
2. a) Prove the necessary conditions for the differentiability of a complex function $f(z) = u(x, y) + i v(x, y)$. State the sufficient conditions also. 7

- b) Determine the analytic function whose real part is

$$x^3 - 3xy^2 + 3x^2 - 3y^2 + 2x + 1. \quad 7$$

3. a) Define residue of a function $f(z)$ at its singularity z_0 . Assuming Laurent's expansion of $f(z)$ in the neighbourhood of z_0 , find its residue at z_0 . 7

- b) Show that

$$\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)} dz = 4\pi(\pi+1)i,$$

where C is the circle, $|z| = 3$.

7



4. a) Find Newton's formula for forward interpolation in terms of x . 8
 b) From the table given below, calculate $\varphi(1.2)$ correct to two decimal places : 6

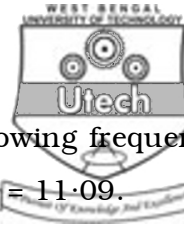
$x :$	0	1	2	3	4
$\varphi(x) :$	1.00	1.50	2.20	3.10	4.60

5. a) Find the third approximate value of the positive root of the equation $x^3 - 2x + 5 = 0$ correct to four decimal places using the Newton-Raphson method. 7
 b) If $n(A)$ and $n(B)$ denote the number of elements in the finite sets A and B respectively, then prove by using the Venn Diagram that

$$n(A) + n(B) = n(A \cup B) + n(A \cap B). \quad 7$$

6. a) Find the extremal of the function $\int_0^4 y \sqrt{1+y'^2} dx$. 6
 b) Discuss Lagrange's Multiplier method to solve constrained problems of optimization. 8

7. a) A committee of 4 people is to be appointed from 3 officers of the production department, 4 officers of the Purchase department, 2 officers of the Sales department and one Chartered accountant. Find the probability of forming the committee in the following manner :
 i) There must be one from each category.
 ii) It should have at least one from the purchase department.
 iii) The chartered accountant must be in the committee. 6



- b) Find the missing frequencies in the following frequency distribution when it is known that mean = 11.09. 8

Class limits :	9.3-9.7	9.8-10.2	10.3-10.7	10.8-11.2
Frequency :	2	5	f_3	f_4

11.3-11.7	11.8-12.2	12.3-12.7	12.8-13.2	Total
14	6	3	1	60

8. Write short notes on any *three* of the following : 14

- Couchy's fundamental theorem for a complex function
- Method of 'Regula Falsi'
- A finite set having n elements has 2^n sub-sets.
- Axiomatic definition of probability.
