Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/M.Tech(ECE)/SEM-1/MCE-101/2011-12

2011

ADVANCED ENGINEERING MATHEMATICS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Answer Question No. 1 and any *four* from the rest.

1. Answer the following questions : $7 \times 2 = 14$

   i) Obtain $P\left(\bar{A} + B\right)$, $P\left(A + \bar{B}\right)$ in terms of $P(A)$, $P(B)$ and $P(AB)$.

   ii) If X is normal (m, σ), then prove that

   $$P(a < X < b) = \Phi\left(\frac{b-m}{\sigma}\right) - \Phi\left(\frac{a-m}{\sigma}\right).$$

   iii) What is the difference between saddle point and extreme point of a function of two variables ?

   iv) Prove that $P(a < X \leq b) = F(b) - F(a)$.

   v) Let $f(z)$ be analytic in a domain d then show that f is constant if $Re\left\{f(z)\right\}$ is constant in D.

vi) Let $f(z) = \frac{1}{z^2}$ and Γ be the straight line joining the points i and $2 + i$. Show that $\left| \int_{\Gamma} f(z)\, dz \right| \leq 2$.

vii) Prove that $\left(\frac{\Delta^2}{E} \right) x^3 = 6x$.

2. a) If $u = \tan^{-1} \frac{x^3 + y^3}{x - y}$, find the value of

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \, \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}.$$ 5

b) Investigate the continuity of the following functions at the given points :

i) $f(x, y) = \frac{xy}{x^2 + y^2}$, $(x, y) \neq (0, 0)$.

$= 0$, $(x, y) = (0, 0)$ at $(0, 0)$. 3

ii) $f(x, y) = x \sin \frac{1}{y} + y \sin \frac{1}{x}$ if $xy \neq 0$

$= 0$, $xy = 0$ at $(0, 0)$ 3

c) State the geometrical interpretation of partial derivative of a function of two independent variables. 3

3. a) A top-open rectangular box has capacity 32 cft. Find the dimension of the box so that the total surface area is minimum. 5

b) There are two identical urns containing 4 white and 3 red balls, 3 white and 7 red balls. An urn is chosen at random and a ball is drawn from it. Find the probability that it is from the first urn if the ball drawn is white. 5

c) Two random variables X, Y have the least square regression lines with equations $3x + 2y - 26 = 0$ and $6x + y - 31 = 0$. Find $E(X)$, $E(Y)$ and $\rho(X, Y)$. 4

4. a) One card is selected at random from 100 cards numbered 00, 01, 02, ... , 98, 99. Suppose n_1 and n_2 are the sum and product respectively of the digits on the selected card. Find $P\{n_1 = i \mid n_2 = 0\}$. 5

b) If $F(x, y, z) = 0$, prove that

$$\left(\frac{\partial x}{\partial y}\right)_z \times \left(\frac{\partial y}{\partial z}\right)_x \times \left(\frac{\partial z}{\partial x}\right)_y = -1. \qquad 4$$

c) If $u = f\left(\frac{y-x}{xy}, \frac{zx}{xz}\right)$ then prove that

$$x^2 \frac{\partial u}{\partial x} + y^2 \frac{\partial u}{\partial y} + z^2 \frac{\partial u}{\partial z} = 0. \qquad 5$$

5. a) State Cauchy fundamental theorem. 1

b) Evalute $\int_{|z|=1} \frac{dz}{z+2}$ and deduce that

$$\int_0^{\pi} \frac{1 + 2\cos\theta}{5 + 4\cos\theta}\, d\theta = 0. \qquad 1 + 4$$

c) Find the bilinear transformation which transforms the points $z = 1, -i, -1$ into the points $\omega = 0, i, \infty$ and show that the area of the circle $|z| = 1$ is transformed into the half plane above the real axis in ω-plane. 4 + 4

6. a) Solve the differential equation,

$$u_{x+2} - 7u_{x+1} + 12u_x = \cos x$$ 4

b) Use Runge-Kutta method of order 2 to calculate $y(0{\cdot}2)$ for the equation $\frac{dy}{dx} = x + y^2$, $y(\sigma) = 1$. 5

c) Find the maximum and minimum values of y from the following : 5

x :	0	1	2	5
y :	2	3	12	147

7. a) Find the cube root of 10 up to 5 significant figures by Newton-Raphson method. 6

b) State and prove Baye's theorem (in probability). 8