Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/M.TECH(ECE/EE)/SEM-1/MCE/EMM-101/2011-12

2011

ADVANCED ENGINEERING MATHEMATICS

Time Allotted : 3 Hours *Full Marks :* 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

   i) Given that $P(A) = 1/3$, $P(B) = 1/4$, $P(A/B)=1/6$. Then $P(B/A)$ is equal to

      a) 1/8 b) 1/7

      c) 1/6 d) none of these.

   ii) A coin is tossed six times. The probability of obtaining heads and tails alternately is

      a) 1/16 b) 1/32

      c) 1/8 d) none of these.

   iii) The *p.d.f* of a variable is defined as $f(x) = cx(2-x)$, $0 < x < 2$. Then c is

      a) 3/4 b) 2/3

      c) 1/3 d) none of these.

   iv) If $f(z) = \bar{z}$, then $f'(0)$ is

      a) 1 b) – 1

      c) 0 d) does not exist.

v) If $f(z) = u(x, y) + iv(x, y)$ then $f'(z)$ is

a) $\frac{\partial v}{\partial y} - i\frac{\partial u}{\partial y}$ b) $\frac{\partial u}{\partial x} - i\frac{\partial v}{\partial x}$

c) $\frac{\partial u}{\partial y} + i\frac{\partial v}{\partial x}$ d) $\frac{\partial u}{\partial x} + i\frac{\partial v}{\partial y}$.

vi) Evaluate $\oint_C \frac{2}{z-\alpha} dz$, where C is the circle whose equation is $|z-\alpha| = \rho$

a) $4\pi i$ b) $2\pi i$

c) $\frac{\pi i}{2}$ d) πi.

vii) The order of he pole $z = 0$ of function $\frac{\sin z}{z^3}$ is

a) 1 b) 2

c) 3 d) 4.

viii) Geometrically the Lagrange interpolation formula for two points of Interpolation represents a

a) parabola b) straight line

c) circle d) none of these.

ix) Can you apply Newton's forward and backward interpolation formula for Unequal spaced interpolation points ?

a) No

b) Yes

c) Yes, when the number of points is greater than three

d) None of these.

x) If tie occurs in selecting the departing vector, then the next solution must be

a) degenerate b) generate

c) iteration d) none of these.

xi) A linear programming problem possesses a finite optimal solution iff there exist feasible solutions

a) both primal and dual problems

b) only primal problem

c) only dual problem

d) none of these.

xii) If for a binomial distribution b (x; n, p) n = 4 and also $P(x = 2) = 3P\ (x = 3)$, the value of P is

a) 1/3 b) 1/2

c) 1/4 d) none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. 3 × 5 = 15

2. If $u - v = (x - y)(x^2 + 4xy + y^2)$ and f (z) = $u + iv$ is an analytic function of $z = x + iy$, find f (z) in terms of z.

3. Evaluate $\frac{1}{2\pi i} \int_C \frac{e^{zt}}{(z^2+1)^2} dz$, if $t > 0$ and C is the circle $|\ z\ | = 3$.

4. Using Cauchy's Residue theorem, prove that

$$\int_0^{2\pi} \frac{\sin^2\theta}{a + b\cos\theta}\, d\theta = \frac{2\pi}{b^2}[a - \sqrt{a^2 - b^2}];\ a > 0,\ b > 0.$$

5. Find the value of the constant k such that

$f(x) = kx(1 - x)$, $0 < x < 1$
$= 0$, elsewhere

is a possible density function and compute $P(x > 1/2)$. Also find E (X).

6. Prove that $-1 \le r_{xy} \le 1$, where r_{xy} is correlation coefficient of x and y.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. 3 × 15 = 45

7. a) Let (x , y) and (u, v) represent two sets of bivariate data such that $u = ax + b$ and $v = cy + d$, then prove that $r_{uv} = \frac{ac}{|a||c|} r_{xy}$; where a, b, c, d are constants.

b) Calculate the coefficient of correlation and obtain the lines of regression for the following data :

x :	1	2	3	4	5	6	7	8	9
y :	9	8	10	12	11	13	14	16	15

Obtain an estimate of y which should corresponds on the average to $x = 6{\cdot}2$.

8. a) Find mean and variance of Binomial and Normal distributions.

b) If X is normally distributed with mean 3 and s.d. 2, find c such that $P(X > c) = 2P(X \le c)$. Given $\int_{-\infty}^{0.43} \phi(t)dt = 1/3$

9. a) Apply Newton's forward interpolation formula to find $f'(0)$ and $f''(0)$ (correct up to 3 decimal places) from the following data :

x :	0	0.4	0.8	1.2
$f(x)$:	0	0.493	2.022	4.666

b) Determine the eigenvalues and the corresponding eigenvectors of the matrix $A = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{bmatrix}$

10. a) Solve the following L.P.P. :

Minimize Z = 4x + 8y + 3z

$s.t.\ x + y \ge 2$

$2x + z \ge 5$

$x, y, z \ge 0.$

b) Use dual simplex method to solve the following L.P.P. :

Minimize $Z = x + 2y + 2z$

$s.t\ 4x - 5y + 7z \le 8$

$2x - 4y + 2z \ge 2$

$x - 3y + 2z \le 2$

$x, y, z \ge 0$