Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/M.Tech(ECE)/SEM-1/MM(EC)-901/2009-10

2009

ADVANCED ENGINEERING MATHEMATICS & NUMERICAL TECHNIQUES

Time Allotted : 3 Hours   Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives of the following :

10 × 1 = 10

i) The partial differential equation $f_{xx} - 2f_{xy} = 0$ is

a) elliptic   b) parabolic

c) hyperbolic   d) none of these.

ii) $P_o(x) = 1$.

a) True   b) False.

iii) Milne's method is a predictor-corrector method.

a) True   b) False.

iv) Crank-Nicolson method is known as an implicit scheme.

a) True   b) false.

v) Euler's method is the Runge-Kutta method of the first order.

a) True   b) False.

vi) Shooting method is used to find the solution of intitial value problems.

a) True b) False.

vii) $\int_{-1}^{1} P_m(x)\, P_n(x)\, dx\ (m \neq n)$ is equal to

a) 0 b) 1

c) -1 d) none of these.

viii) Order of the differential equation

$$\frac{d^2y}{dx^2} = \phi\left(x, y, \frac{dy}{dx}\right) \text{ is}$$

a) 1 b) 2

c) 3 d) none of these.

ix) $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + (x^2 - p^2)\, y = 0$ is known as Legendre's differential equation.

a) True b) False.

x) $(1 - x^2) \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} + n(n+1)\, y = 0$ is known as the Bessel's differential equation.

a) True b) False.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Find the Laplace transform of t^n, n is positive but not necessarily an integer.

3. Find the Fourier transform of the function $f(x)$,

$$\text{if } f(x) = \begin{cases} e^{iwx}, & a < x < b \\ 0, & x < a, x > b \end{cases}$$

4. Show that the shortest distance between two points in a plane is a straight line.

5. Express x^5 in terms of Legendre polynomials.

6. Prove that $\frac{d}{dx}\{x^p J_p(x)\} = x^p J_{p-1}(x)$.

7. Express $J_4(x)$ in terms of $J_0(x)$ and $J_1(x)$.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

8. a) Classify the equation

$$U_{xx} + 4U_{xy} + (x^2 + 4y^2) U_{yy} = \sin xy.$$

b) Write down the finite difference approximation for the partial derivatives $\frac{\delta^2 u}{\delta x^2}$, $\frac{\delta^2 u}{\delta y^2}$.

c) Derive Schmidt's formula to find the solution of one-dimensional heat equation $U_{xx} - aU_t = 0$. 5 + 5 + 5

9. a) Find the Fourier series generated by the periodic function $|x|$ of period 2π. Hence prove that $1 + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots\ldots\ldots = \frac{\pi^2}{8}$.

b) Evaluate the inverse Laplace transform of

$$\frac{4}{s^2} + \frac{(\sqrt{s} - 1)^2}{s^2} - \frac{5}{3s + 4}.$$ 8 + 7

10. a) Use Laplace transform to solve

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0, \text{ given that } y = 1 \text{ and } \frac{dy}{dx} = 1$$

when $x = 0$.

b) Use convolution theorem to find $L^{-1}\left\{\frac{s}{(s^2 + a^2)^2}\right\}$.

8 + 7

11. a) If the Fourier transform of $F(x)$ is $f(s)$, then show that the Fourier transform of $F'(x)$, the derivative of $F(x)$ is $(-is)f(s)$.

b) Find the Fourier sine and cosine transforms of the function $F(x) = x^{m-1}$.

12. a) Derive Euler's equation to determine a function $f(x)$, for which the value of the definite integral

$$I = \int_{x_1}^{x_2} f\left(x, y, \frac{dy}{dx}\right) dx$$ is a maximum or minimum

under two end conditions $y_1 = f(x_1)$ and $y_2 = f(x_2)$.

b) If the integral $I = \int_{x_1}^{x_2} \frac{y'^2}{y^2} dx$ is an extremum

show that y can be expressed in the form

$y = ae^{bx}$, a & b are constants. 9 + 6

13. a) From the Rodrigue's formula $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}(x^2-1)^n$

prove that $P_n'(x) = xP'_{n-1}(x) + nP_{n-1}(x)$.

b) Using the Runge-Kutta method of fourth order find $y(0 \cdot 2)$ in steps of $(0 \cdot 1)$, given that

$\frac{dy}{dx} = 1 + xy, \; y(0) = 2.$