



Name : .....

Roll No. : .....

Invigilator's Signature : .....

**CS/M.Tech(ECE)/SEM-1/MM(EC)-901/2009-10  
2009**

**ADVANCED ENGINEERING MATHEMATICS &  
NUMERICAL TECHNIQUES**

Time Allotted : 3 Hours

Full Marks : 70

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

**GROUP – A**

**( Multiple Choice Type Questions )**

1. Choose the correct alternatives of the following :

$$10 \times 1 = 10$$

- i) The partial differential equation  $f_{xx} - 2f_{xy} = 0$  is
  - a) elliptic
  - b) parabolic
  - c) hyperbolic
  - d) none of these.
- ii)  $P_0(x) = 1$ .
  - a) True
  - b) False.
- iii) Milne's method is a predictor-corrector method.
  - a) True
  - b) False.
- iv) Crank-Nicolson method is known as an implicit scheme.
  - a) True
  - b) false.
- v) Euler's method is the Runge-Kutta method of the first order.
  - a) True
  - b) False.

- GROUP – B**

Answer any *three* of the following.  $3 \times 5 = 15$

- $$\text{if } f(x) = \begin{cases} e^{i\omega x} & , a < x < b \\ 0 & , x < a, x > b \end{cases}$$



5. Express  $x^5$  in terms of Legendre polynomials.
6. Prove that  $\frac{d}{dx} \{ x^p J_p(x) \} = x^p J_{p-1}(x)$ .
7. Express  $J_4(x)$  in terms of  $J_0(x)$  and  $J_1(x)$ .

### GROUP – C

#### ( Long Answer Type Questions )

Answer any *three* of the following.  $3 \times 15 = 45$

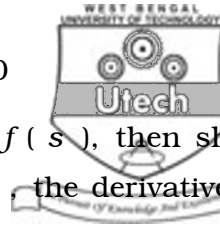
8. a) Classify the equation  

$$U_{xx} + 4U_{xy} + (x^2 + 4y^2) U_{yy} = \sin xy.$$
- b) Write down the finite difference approximation for the partial derivatives  $\frac{\partial^2 u}{\partial x^2}$ ,  $\frac{\partial^2 u}{\partial y^2}$ .
- c) Derive Schmidt's formula to find the solution of one-dimensional heat equation  $U_{xx} - aU_t = 0$ .  $5 + 5 + 5$
9. a) Find the Fourier series generated by the periodic function  $|x|$  of period  $2\pi$ . Hence prove that  

$$1 + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}.$$
- b) Evaluate the inverse Laplace transform of  

$$\frac{4}{s^2} + \frac{(\sqrt{s} - 1)^2}{s^2} - \frac{5}{3s + 4}.$$
  $8 + 7$
10. a) Use Laplace transform to solve  

$$\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + 2y = 0, \text{ given that } y = 1 \text{ and } \frac{dy}{dx} = 1 \text{ when } x = 0.$$
- b) Use convolution theorem to find  $L^{-1} \left\{ \frac{s}{(s^2 + a^2)^2} \right\}.$   $8 + 7$



11. a) If the Fourier transform of  $F(x)$  is  $f(s)$ , then show that the Fourier transform of  $F'(x)$ , the derivative of  $F(x)$  is  $(-is)f(s)$ .

b) Find the Fourier sine and cosine transforms of the function  $F(x) = x^{m-1}$ .

12. a) Derive Euler's equation to determine a function  $f(x)$ , for which the value of the definite integral

$$I = \int_{x_1}^{x_2} f\left(x, y, \frac{dy}{dx}\right) dx \text{ is a maximum or minimum}$$

under two end conditions  $y_1 = f(x_1)$  and  $y_2 = f(x_2)$ .

b) If the integral  $I = \int_{x_1}^{x_2} \frac{y'^2}{y^2} dx$  is an extremum

show that  $y$  can be expressed in the form

$$y = ae^{bx}, \text{ } a \text{ \& } b \text{ are constants.}$$

9 + 6

13. a) From the Rodrigue's formula  $P_n(x) = \frac{1}{2^n n!}$

$$\frac{d^n}{dx^n} (x^2 - 1)^n$$

prove that  $P_n'(x) = xP_{n-1}'(x) + nP_{n-1}(x)$ .

b) Using the Runge-Kutta method of fourth order find  $y(0.2)$  in steps of  $(0.1)$ , given that

$$\frac{dy}{dx} = 1 + xy, y(0) = 2.$$

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