Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS/M.Tech (ECE) VLSI/SEM-1/MH-901/2009-10

2009

ADVANCED ENGINEERING MATHEMATICS

Time Allotted : 3 Hours Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A
(Objective Type Questions)

1. Answer any *five* of the following : 5 × 2 = 10

i) Find the value of $\oint_C \frac{e^z\,dz}{z-2}$, where C : $|z - 2| = 4$.

ii) Find the residue of $f(z)=\frac{2+3\sin\pi z}{z(z-1)^2}$.

iii) Evaluate $\int_\gamma \frac{\sin z}{z}dz$, in $|z|=1$.

iv) If A, B are independent events then prove that $\overline{A}, \overline{B}$ are also independent.

v) If X is normally distributed with zero mean and unit variance, find the variance of $E\left(X^2\right)$.

vi) Assuming root lies in [1, 5], what will be the third (at the end of 3 iterations) iterative value of the root of $te^{-t} - 0 \cdot 3 = 0$ using the bisection method.

vii) Prove that $\Delta \cdot \nabla = \Delta - \nabla$.

viii) If $\frac{dy}{dx} = x + y$ and y (0) = 1, then find y (0·01) according to Euler's method, where h = 0·01.

GROUP – B
(Short Answer Type Questions)

Answer any *three* of the following. 3 × 5 = 15

2. For n events $A_1, A_2, \ldots\ldots, A_n$ $P\left(\bigcup_{i=1}^{n} A_i\right) \leq \sum_{i=1}^{n} P\left(A_i\right)$.

3. The integers x and y are chosen at random with replacement from nine natural numbers 1, 2, ... , 9. Find the probability that $\left|x^2 - y^2\right|$ is divisible by 2.

4. Show that $f(z)=|z|^2$ is continuous everywhere but it is nowhere differentiable except origin.

5. Evaluate $\int_{|z|=3} \frac{z+1}{z^2-2z}\,dz$.

6. Write an algorithm to solve an transcendental equation using Newton-Raphson method.

7. If $\frac{dy}{dx}=x^2+y$, $y(0)=1$, $h=0{\cdot}1$, find $y(0{\cdot}2)$ using Euler's modified method.

GROUP – C
(Long Answer Type Questions)
Answer any *three* of the following. $3 \times 15 = 45$

8. a) State and prove Bayes theorem.

   b) A missile was fired at two targets T_1 and T_2 such that probability of hitting T_1 is p_1 and that of T_2 is p_2. Find the probability of hitting T_1 on the hypothesis that T_2 was not hit.

c) A man seeks advice regarding one of two possible course of actions from three advisors, who arrive at their recommendation independently. He follows the recommendations of the majority. The probability that the individual advisors are wrong, 0·1, 0·05 and 0·05 respectively. What is the probability that man takes incorrect advice.

9. a) The probability density function of a ramdon variable X is $f(x) = k(x-1)(2-x)$ for $1 \le x \le 2$.

Determine,

i) the value of the constant k

ii) the distribution function $F(x)$

iii) $P\left(\frac{5}{4} \le X \le \frac{3}{2}\right)$.

b) Show that the function $f(x) = |x|$, when $-1 < x < 1$.

$= 0$, elsewhere.

is a possible p.d..f and find the corresponding distribution function.

c) If X is normally distributed with mean 3 and s.d. 2, find c such that $P(X > c) = 2P(X \leq c)$, where $\int_{-\infty}^{0\cdot43} \phi(t)\,dt = \frac{1}{3}$.

10. a) State and prove Cauchy's Integral formula.

b) If f is analytic in $|z| \leq 1$ and $|f(z)| \leq 1$, $\forall z$ in $|z| \leq 1$. Show that $|f'(0)| \leq 1$.

c) Let $p(z)$ be a polynomial where $z \in \mathbb{C}$. Prove that $p(\mathbb{C}) = \mathbb{C}$. Is the result true for any entire function ? Justify your answer.

11. a) Find the bilinear transformation which maps the points $z = 1, 0, -1$ onto the points $w = i, 0, -i$. Also find the fixed points of the transformation.

b) Evaluate the following integral by contour integration :

$$\int_{-\infty}^{\infty} \frac{dx}{(1+x^2)^3}.$$

c) Find the Laurent series expansion of $f(z)=\frac{1}{z^2(1-z)}$ in the regions (i) $0 < |z| < 1$, (ii) $1 < |z| < \infty$.

12. a) Prove the condition of convergence for Newton-Raphson method to solve a transcendental equation $\left|f(x).f''(x)\right| \le \left(f'(x)\right)^2$.

b) The values of sin x are given below for different values of x. Form a difference table and from this table find the value of sin 32°.

x (in degree)	30	35	40	45	50	55
$y = \sin x$	0·5	0·5736	0·6437	0·7071	0·7660	0·8192

c) Find a root of the equation $xe^x = \sin x$ using Regula Falsi method correct up to 3 decimal places.

13. a) Write an algorithm to estimate the function value using Newton's backward interpolations.

b) Find a root of the equation $x^3 = 2$ using fixed point method correct upto 3 decimal places.

c) If $\frac{dy}{dx} = 3x + \frac{y}{2}$, y (0) = 1, h = 0·1, find y (0·2) using Runge-Kutta method of 4th order.