Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/M.TECH (ECE)/SEM-1/MCE-101/2012-13

2012

ADVANCED ENGINEERING MATHEMATICS

Time Allotted : 3 Hours　　　　　　　　　　　　Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer Question No. **1** and any *four* from the rest.

1. a) Find the critical point of the function $f(x, y) = xy$. 2

   b) Show that another form of Euler-Lagrange equation is $d/dx\,[\,f - y'\,\partial f/\partial y'\,] - \partial f/\partial x = 0$. 3

   c) Evaluate $\int_C \sin 6z/(z - \pi/6)^3\, dz$, if $C : |z| = 1$. 3

   d) Find the Laurents Expansion for

      $f(z) = (z-2)(z+2)/(z+1)(z+4)$,

      when $1 < |z| < 4$. 3

   e) If A and B are two mutually independent events then show that A' and B' are also mutually independent. 3

2. a) State and prove Bayes' theorem. 8

b) In a bolt factory, machines A, B, C manufacture 25, 35 and 40 per cent of the total respectively. Of this output 5, 4 and 2 per cent are defective bolts. A bolt is drawn at random from the product and is found defective. What is the probability that it was manufactured by C ? 6

3. a) Show that the shortest wave joining two fixed points is a straight line. 7

b) If $f(z)$ be analytic within and on a simple closed contour C, then the point giving the maximum of the $|f(z)|$ can be on the boundary c and within it prove that. 7

4. a) Use Runge's method to find an appropriate value of y when $x = 0{\cdot}8$ given that

$dy / dx = (x + y)^{1/2}$ when $y(0{\cdot}4) = 0{\cdot}41$. 6

b) Given $dy/dx = x^2(1 + y)$ and $y(1) = 1$,

$y(1{\cdot}1) = 1{\cdot}233$, $y(1{\cdot}2) = 1{\cdot}548$,

$y(1{\cdot}3) = 1{\cdot}979$, Evaluate $y(1{\cdot}4)$ by Milne's predictor corrector method. 8

5. a) Find the optimum value of $f(x, y) = x^2 y^2$ subject to the condition $x + y = 1$ using Lagrange's multiplier method. 7

b) Show that, if $f(z)$ is continuous function in a simply connected domain D and if $\int_c f(z)\,dz = 0$, where c is any rectifiable closed Jordon curve in D then $f(z)$ is analytic in D. 7

6. a) Show that the necessary condition for

$$\int_{x_1}^{x_2} f(x, y, y') \, dx$$ to be an extremum is

$\partial f/\partial y - d/dx\,(\partial f/\partial y') = 0$. 7

b) Find the mean and variance of binomial distribution having parameters n and p. 7

7. a) Find the extremals of the following functionals

i) $$\int_{x_0}^{x_1} (x + y')\, y' \, dx$$

ii) $$\int_{x_0}^{x_1} (y'^2 / x^3) \, dx.$$ 7

b) Solve the following difference equations :

i) $(\Delta^2 - 3\Delta + 2)\, y_x = 0$

ii) $U_{n+3} - 3U_{n+2} + 4U_n = 0$. 7

8. a) Evaluate $\int_0^{2\pi} e^{-\cos\theta} (\cos(n\theta + \sin\theta))\, d\theta$ where n is a positive integer. 8

b) If $f(z) \to 0$ uniformly as $|z| \to \infty$ and $f(z)$ is meromorphic in the upper half plane then show that

$$\underset{R \to \infty}{Lt} \int_{c_R} e^{imz} f(z)\, dz = 0 \quad (m > 0)$$

where CR denotes semicircle $|z| = R$, $I(z) > 0$. 6