Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/M.TECH(ECE)/SEM-1/MCE-101/2012-13

2012

ADVANCED ENGINEERING MATHEMATICS

Time Allotted : 3 Hours   *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words
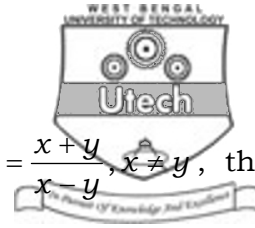
as far as practicable.

Answer Q. No. 1 and any *four* from the rest.

1. a) Evaluate $\oint_C \dfrac{z}{z^2 - 25}\, \mathrm{d}z$ on the circle $C : \mid z - 1 \mid\, = 2$.

    b) If $f(x, y) = 0$, prove that

    $$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = -\frac{\{f_{xx}f_y^2 - 2f_x f_y f_{xy} + f_{yy}f_x^2\}}{f_y^3},\ \text{provided}\ f_y \neq 0.$$

    c) Find the positive root of the equation $x^3 - 8x - 4 = 0$, correct up to three significant figures by Newton-Raphson method.   4 + 5 + 5

2. a) Show that for the function $f(x,y) = \dfrac{x+y}{x-y}, x \neq y$, the

repeated limits exist and $\displaystyle \lim_{y \to 0} \lim_{x \to 0} \frac{x+y}{x-y} = -1$ and

$\displaystyle \lim_{x \to 0} \lim_{y \to 0} \frac{x+y}{x-y} = -1$. But the double limit $\displaystyle \lim_{\substack{x \to 0 \\ y \to 0}} \frac{x+y}{x-y}$

does not exist.

b) If $u = \sin^{-1} \sqrt{\dfrac{x^{1/3} + y^{1/3}}{x^{1/2} + y^{1/2}}}$, then verify whether the

following identity is true :

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{12}\left(\frac{13}{12} + \frac{\tan^2 u}{12}\right)$$

(3 + 4) + 7

3. a) Find the maximum and minimum values of the function

$f(x,y) = x^3 + y^3 - 3axy$

b) Find the point upon the plane $ax + by + cz = p$ at which

the function $f(x,y,z) = x^2 + y^2 + z^2$ has a minimum

value and find this minimum. Use Lagrange's multiplier

method. 6 + 8

4. a) Determine the analytic function whose real part is

$2x - 2xy$.

b) Evaluate $\displaystyle \oint_C \frac{\cos^3 z}{\left(z - \frac{\pi}{4}\right)^3} dz$ by Cauchy's Integral formula,

where C is the circle $|z| = 1$. 7 + 7

5. a) Find the singular points and determine the nature of the singularities in each case :

(i) $\dfrac{e^z}{(z-1)^3}$ and (ii) $\dfrac{z}{e^{z-2}}$.

b) Find the residues of the function $\dfrac{\cos \pi z}{(z-a)^2}$.   (3 + 4) + 7

6. a) Find the positive root of the equation $x^3 - 3x - 1 \cdot 06 = 0$, correct up to two significant figures by the Bisection method.

b) Using Lagrange's interpolation formula find the value of $f(x)$ at $x = 5$ from the following data :

x	3	7	9	12
$f(x)$	11	17	24	30

7 + 7

7. a) Use Euler's method to compute $y(0 \cdot 1)$, where $\dfrac{dy}{dx} = \dfrac{y-x}{y+x}$, given $y = 1$, when $x = 0$ with step length $h = 0 \cdot 02$.
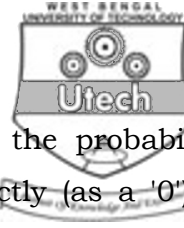
b) Using Gauss Elimination method solve

$x_1 + 3x_2 + 2x_3 = 5$

$2x_1 - x_2 + x_3 = -1$

$x_1 + 2x_2 + 3x_3 = 2$ 7 + 7

8. a) For a binary communication channel, the probability that a transmitted '0' is received correctly (as a '0') is 0·95 and the probability that a transmitted '1' is received correctly (as a '1') is 0·90. The probability that '0' is transmitted is 0·4. If '1' is received then find the probability that '1' was transmitted.

   b) Two cells are picked up randomly from a chessboard. What is the probability that they share a side ?   7 + 7

9. When a moving car applies brake, it covers some distance before stopping. The following data was collected on the relation between the initial speed x and the distance covered y, based on the same car and the highest level of brake was applied in each case.

Initial speed (x)	20	20	30	30	30	40	40	50	50	60
Stopping Distance (y)	16·3	26·7	39·2	63·5	51·3	98·4	65·7	104·1	155·6	217·2

Both theory and the above data (after plotting) suggest that there is a linear relation between x and $\sqrt{y}$.

Use non-linear regression to find the relation between x and y. Also estimate the stopping distance if the initial speed is 70.   11 + 3

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