Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/M.Tech(ECE)/SEM-1/MCE-101/2012-13

2012

ADVANCED ENGINEERING MATHEMATICS

Time Allotted : 3 Hours   *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A
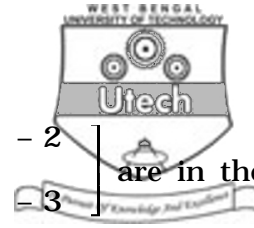
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following :

$10 \times 1 = 10$

i) The matrix $A = \begin{bmatrix} 2 & 3 \\ 3 & 7 \end{bmatrix}$ is

a) positive definite   b) negative definite

c) positive semi-definite   d) negative semi-definite.

ii) The backward difference formula for the first derivative of $y(x)$ can be written as

a) $\dfrac{y_{i-1} - y_i}{h}$ b) $\dfrac{y_i - y_{i-1}}{h}$

c) $\dfrac{y_{i+1} - y_i}{h}$ d) $\dfrac{y_{i-1} + y_i}{h}$.

iii) The eigenvalues of the matrix $\begin{bmatrix} 1 & -2 \\ 2 & -3 \end{bmatrix}$ are in the

location of

a) $|\lambda + 2| \le 1, \ |\lambda - 1| \le 2$

b) $|\lambda - 3| \le 2, \ |\lambda + 1| \le 1$

c) $|\lambda + 3| \le 2, \ |\lambda - 1| \le 2$

d) none of these.

iv) If $a = 2 + i\,3$ and $b = 3 = i\,2$, then

a) $a = b$ b) $a < b$

c) $a > b$ d) none of these.

v) If $f(z) = -\dfrac{1}{z - 1} - 2\left[\,1 + (z - 1) + (z - 1)^2 + \ldots\ldots\,\right]$

then the residue of $f(z)$ at $z = 1$ is

a) 1   b) -1

c) 0   d) none of these.

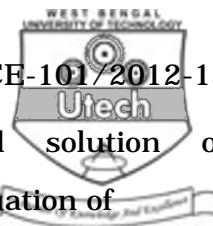vi) The value of $\displaystyle\int_C \dfrac{dz}{z - 1}$, C being $|z| = 2$ is

a) 2π b) -2π

c) $2\pi i$ d) $-2\pi i$.

vii) Runge-Kutta method for numerical solution of differential equations is applicable to equation of

    a)   first order first degree

    b)   second order first degree

    c)   first order higher degree

    d)   none of these.

viii) If events are mutually exclusive, then

    a)   their probabilities are less than 1

    b)   their probabilities sum to 1

    c)   both events cannot occur at the same time

    d)   both of them contain every possible outcome of an experiment.

ix) The probability of getting more than 4 in rolling a die is

    a) $\dfrac{1}{6}$ b) $\dfrac{1}{3}$

    c) $\dfrac{1}{2}$ d)   1.

x) The value of correlation coefficient

    a)   depends on the origin

    b)   depends on the unit of scale

    c)   depends on both origin and unit of scale

    d)   is independent with respect to origin and unit of scale.

xi) When two or more events occur in connection with each orther, their simultaneous occurrence is called a(n)

    a)   exhaustive event   b)   compound event

    c)   equally likely event   d)   simple event.

xii) The standard deviation of the binomial distribution is

    a) np b) $\sqrt{np}$

    c) npq d) $\sqrt{npq}$.

GROUP – B
(Short Answer Type Questions)
Answer any *three* of the following.   3 × 5 = 15

2. Evaluate $\displaystyle\int_C \frac{2z^2 + z + 1}{(z^2 - 1)(z + 3)}\, dz$ where $|z| = 2$.

3. Apply Newton-Raphson method to find the root of the equation $3x \cos x - 1 = 0$ in the vicinity of $x = 0.61$.

4. Show that the function

$$f\left(x_1, x_2 \right) = 9 - 4x_1 - 2x_2 + x_1^2 + 7x_1 x_2 + x_2^2 + x_1^3 + x_2^3 + 3x_1^2 x_2^2$$

has a minimum at (1, – 1).

5. Fit a straight line of Y on X from the following data :

x	0	1	2	3	4	5	6
y	2	1	3	2	4	3	5

6. In a Poisson distribution $3P\left(x = 2 \right) = P\left(x = 4 \right)$. Find $P\left(x = 3 \right)$.

GROUP – C
(Long Answer Type Questions)
Answer any *three* of the following.　　3 × 15 = 45

7. a) Expand $f (z) = \dfrac{1}{z^2 - 5z + 6}$ in Laurent series valid in

　　$2 < |z| < 3$ 　　　　　　　　　　　　　　5

b) Find Taylor series expansion of $f (z) = \dfrac{2z^3 + 1}{z^2 - 5z + 6}$ in

　　the neighbourhood of $z = i$. 　　　　　　　5

c) Using Runge-Kutta method, find an approximate value

　　of y for $x = 0.2$ if $\dfrac{dy}{dx} = x + y^2$, $y (0) = 1$ by taking

　　the step length $h = 0.1$. 　　　　　　　　5

8. a) Find the extremum of the function

　　$$f \left(x_1, x_2 \right) = 5x_1 x_2 - 2x_1^2 - 4x_2^2 - x_1 + 3x_2 + 8.$$

　　　　　　　　　　　　　　　　　　　5

b) If $u - v = (x - y) (x^2 + 4xy + y^2)$ and $f (z)\ u + iv$ is

　　an analytic function of $z = x + iy$, find $f (z)$ in terms

　　of z. 　　　　　　　　　　　　　　5

c) If $\lambda = 3$ is an eigenvalue and $\begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$ is the

　　corresponding eigenvector of the matrix

　　$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$, then apply deflation method to

　　find the other eigenvalues of A. 　　　　　5

9. a) Apply Finite Difference method to solve the equation

$$\frac{d^2y}{dx^2} = 4x + 3y, \; y(0) = 0, \; y(1) = 1, \text{ in the range}$$

$0 \le x \le 1$ by taking the mesh length $h = 0.2$.   8

b) Evaluate $\displaystyle\int_C \frac{2 + 3 \sin \pi z}{z(z-1)^2} \, dz$, where C is a square

having vertices at $3 + i\,3$, $3 - i\,3$, $-3 + i\,3$, $-3 - i\,3$.   7

10. a) A special type of wooden beam has a mean breaking strength of 1500 kgs and a standard deviation of 100 kgs. Find the percentage of all such beams whose breaking strength lies between 1450 kgs and 1600 kgs. It is given that the areas under the standard normal distribution curve between $z = 0$ and $z = 0.5$ and $z = 0$ and $z = 1.0$ are 0.1915 and 0.3413 respectively.   8

b) A product consists of two parts A and B. The manufacturing process of each part is such that probability of defect in A is 0.12 and that in B is 0.17. If 100 units are assembled with A and B, how many units of the product are expected to be free from defect ?   7

11. a) (i) Define a probability distribution and explain in brief its classification. 2

   (ii) Define mean and variance of a discrete probability distribution. 2 + 3

   b) What are the assumptions for the study of analysis of variance ? 3

   c) Eight letters to each of which corresponds an envelope are placed in the envelope at random. What is the probability that all letters are not placed in their right envelopes ? 5

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