Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS/M.Tech (ECE)/SEM-1/MCE-103/2010-11

2010-11

ADVANCED DIGITAL SIGNAL PROCESSING

Time Allotted : 3 Hours *Full Marks :* 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer Question No. 1 and any *four* from the rest.

1. Answer *all* questions : 7 × 2

a) Do you compute energy and power of a discrete signal simultaneously ? Justify your answer with an example.

b) Define the necessary and sufficient condition that must be satisfied for the digital filter to be causal and realizable.

c) What are the significance of the position of poles and zeros to design a filter in *Z*-domain ?

d) Among the following systems point out which one has minimum, maximum and mixed phase :

i) $H_1(z) = 6 + z^{-1} - z^{-2}$

ii) $H_2(z) = 1 + 5z^{-1}/3 - 2z^{-2}/3$

iii) $H_3(z) = 1 - 5z^{-1}/2 - 3z^{-2}/2$

iv) $H_4(z) = 1 - z^{-1} - 6z^{-2}$.

e) A signal has 1024 samples. How many levels of Haar transform are possible ? What is the significance of fluctuating part in Wavelet transform ?

f) What would be the problem if your system is not a LTI system ?

g) What is Kalman filter and what can it do ?

2. a) The following input-output pairs have been observed during a time-invariant system :

$$x_1[n] = \{1,\ 0,\ 2\} \xleftrightarrow{T} y_1[n] = \{0,\ 1,\ 0,\ 2\}$$

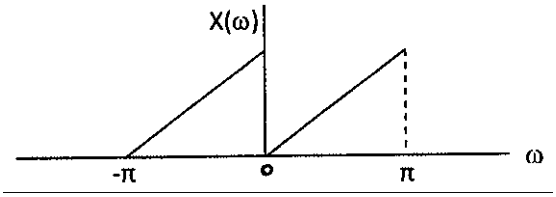
$$x_2[n] = \{0,\ 0,\ 3\} \xleftrightarrow{T} y_2[n] = \{0,\ 1,\ 0,\ 2\}$$

$$x_3[n] = \{0,\ 0,\ 0,\ 1\} \xleftrightarrow{T} y_3[n] = \{1,\ \underset{\uparrow}{2},\ 1\}$$

Can you draw any conclusion regarding the linearity of the system ? What is the impulse response ?

b) Show that a relaxed LTI system is causal if and only if $h[n]=0$ for $n<0$.

c) Determine the signal $x[n]$ if its Fourier transform is as given below :



d) Show that the Fourier transform of the correlation between the signal $x_1[n]$ and $x_2[n]$ is $X_1(\omega)X_2(-\omega)$.

4 + 3 + 4 + 3

3. a) What is the difference between Steady state and Transient response to sinusoidal input signals ?

b) Give an idea how a LTI system can be used as a frequency selective filter.

c) Consider the filter : $y[n]=0\cdot 9\,y[n-1]+bx[n]$.

i) Determine b so that $|H(\omega)|=0$.

ii) Determine the frequency at which $|H(\omega)|=1/\sqrt{2}$.

iii) Is this a LPF, HPF or BPF ? Justify your answer.

d) Knowing $|H(\omega)|^2$, do you determine $H(z)$? Explain.

$2 + 4 + (1\frac{1}{2} + 1\frac{1}{2} + 2) + 3$

4. a) Compute the DFT of the 4-point sequence $x[n]=\{0,\ 1,\ 2,\ 3\}$ using matrix method.

b) Determine the output response $y[n]$, if $h[n]=\{1,\ 1,\ 1\}$; $x[n]=\{1,\ 2,\ 3,\ 1\}$ by using Circular convolution. Under what condition is the result of Circular convolution same as Linear convolution ?

c) Compute a 4-point DFT of a sequence $x[n]=\{0,\ 1,\ 2,\ 3\}$ using DIT-FET. $4 + (3 + 2) + 5$

5. a) What kind of extra features you can get from Wavelet transform over Fourier ?

b) How conservation and compaction of energy are supported in Haar Wavelet transform with respect to the signal f = (4, 6, 10, 12, 8, 6, 5, 5) ?

c) What is the significance of MRA ? Derive an expression of 2 level Haar transform in terms of second average signal $\left(A^2\right)$ and first and second detail signal D^1 and D^2 respectively. 2 + 5 + 7

6. a) For the desired frequency response $H_d(\omega)$, explain the method of designing the linear phase filter of odd length by frequency sampling method with symmetric unit impulse response.

b) Determine the coefficients of linear phase FIR filter of length 15 and symmetric unit impulse response and a frequency response that satisfies

$H_r(2\pi k/15) = 1$ for $k = 0,\ 1,\ 2,\ 3$

$= 0 \cdot 4$ for $k = 4$

$= 0$ for $k = 5,\ 6,\ 7$. 9 + 5

7. a) Define covariance of two random variables x_1 and x_2.

b) Consider an object falling under a constant gravitational field. Let y (t) denotes the height of object. Show that only the estimated state from the previous time step and the current measurement are needed to compute the estimate of current state.

c) If the state of the recursive estimation is represented by following two parameters.

i) $\hat{X}_{k/k}$, the posterior establishment at time k given observation up to and including at time k, and

ii) $P_{k/k}$, the posterior error covariance matrix (a measure of accuracy of the state estimate),

Explain the two phase 'Predict' and 'Update' of the estimator. 2 + 5 + 7

8. a) Explain the energy density spectrum of a continuous signal $x_a(t)$ sampled at a uniform sampling rate F_S to result the sequence $x[n]$.

   b) Compute energy density spectrum of $x_a(t)$ from its samples $x[n]$. Consider $x[n]$ to be of finite duration. 5 + 9