Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/M.Tech (CSE)/SEM-2/PGCS-201/2012

2012

ADVANCED MATHEMATICS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer any *five* questions. $5 \times 14 = 70$

1. a) If the vector field $\vec{F}$ is conservative, then show that there exist a single valued differentiable scalar function ϕ such that $\vec{F}$ is the gradient of ϕ. 7

   b) If $\vec{\nabla} = 2xyz^3\ \vec{i} + x^2 z^3\ \vec{j} + 3x^2yz^2\ \vec{k} + 10\ \vec{j}$, find $\phi(x, y, z)$ such that $\phi(1, -2, 2) = 4$. 7

2. a) Show that $\vec{A} = \left(6xy + z^3\right)\vec{i} + \left(3x^2 - z\right)\vec{j} + \left(3xz^2 - y\right)\vec{k}$ is irrotational. Find a scalar function ϕ such that $\vec{A} = \vec{\nabla}\phi$. 7

   b) Evaluate $\oint_c \left[(3x + 4y)\,dx + (2a - 3y)\,dy\right]$ where c is a circle of radius is 2 and centre at (0, 0) on the xy-plane. 7

3. a) Find the rank of the matrix $\begin{bmatrix} 1 & -1 & 2 & -1 \\ 4 & 2 & -1 & 2 \\ 2 & 2 & -2 & 0 \end{bmatrix}$. 4

b) If $A = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}$, prove that $A^{-1} = A^{T}$. 4

c) Examine the consistency of the following system of equations and solve, when possible :

$x + 2y - z = 10$

$x - y - 2z = -2$

$2x + y - 3z = 8$ 6

4. a) Show that the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$ satisfy its characteristic equation. Hence find its inverse. 3 + 4

b) Find the eigenvalues and corresponding vectors of the matrix : 7

$$\begin{bmatrix} -3 & 1 & -1 \\ -7 & 5 & -1 \\ -6 & 6 & -2 \end{bmatrix}$$

5. a) Evaluate $\int_0^{\infty} t^3 \, e^{-t} \sin t \, dt$, using Laplace transform. 7

b) Solve using Laplace transform :

$$y''(t) + y(t) = 8 \cos t, \; y(0) = 1, \; y'(0) = -1.$$ 7

6. a) Find the Fourier sine transform of $e^{-|x|}$. Hence show that $\int_0^{\infty} \frac{x \sin mx}{1 + x^2} \, dx = \frac{\pi e^{-m}}{2}$, $m > 0$. 7

b) Solve the equation :

$$\left(D^2 + \frac{1}{x} D \right) y = \frac{12 \log x}{x^2}$$ 7

7. a) Solve the equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial u^2}$, given that $u(0, t) = u(l, t) = 0$. $u(x, 0) = f(x)$ and $\frac{\partial u}{\partial t}(x, 0) = 0$, $0 < x < \lambda$. 7

b) Six coins are tossed 6400 times. Using Poisson distribution find the approximate probability of getting six heads 8 times. 7