

Name :

Roll No. :

Invigilator's Signature :

CS/M.Tech (CSE)/SEM-2/PGCS-201/2012
2012
ADVANCED MATHEMATICS

Time Allotted : 3 Hours

Full Marks : 70

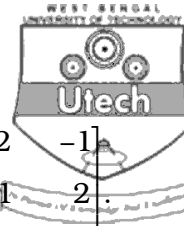
The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

Answer any *five* questions.

5 × 14 = 70

1. a) If the vector field \vec{F} is conservative, then show that there exist a single valued differentiable scalar function ϕ such that \vec{F} is the gradient of ϕ . 7
b) If $\vec{\nabla} = 2xyz^3 \vec{i} + x^2 z^3 \vec{j} + 3x^2 yz^2 \vec{k} + 10 \vec{j}$, find $\phi(x, y, z)$ such that $\phi(1, -2, 2) = 4$. 7
2. a) Show that $\vec{A} = (6xy + z^3) \vec{i} + (3x^2 - z) \vec{j} + (3xz^2 - y) \vec{k}$ is irrotational. Find a scalar function ϕ such that $\vec{A} = \vec{\nabla} \phi$. 7
b) Evaluate $\oint_c [(3x + 4y) dx + (2a - 3y) dy]$ where c is a circle of radius is 2 and centre at $(0, 0)$ on the xy -plane. 7



3. a) Find the rank of the matrix $\begin{bmatrix} 1 & -1 & 2 & -1 \\ 4 & 2 & -1 & 2 \\ 2 & 2 & -2 & 0 \end{bmatrix}$ 4

b) If $A = \frac{1}{3} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}$, prove that $A^{-1} = A^T$. 4

c) Examine the consistency of the following system of equations and solve, when possible :

$$x + 2y - z = 10$$

$$x - y - 2z = -2$$

$$2x + y - 3z = 8 \quad 6$$

4. a) Show that the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$ satisfy its

characteristic equation. Hence find its inverse. 3 + 4

b) Find the eigenvalues and corresponding vectors of the matrix : 7

$$\begin{bmatrix} -3 & 1 & -1 \\ -7 & 5 & -1 \\ -6 & 6 & -2 \end{bmatrix}$$



5. a) Evaluate $\int_0^{\infty} t^3 e^{-t} \sin t \, dt$, using Laplace transform. 7

- b) Solve using Laplace transform :

$$y''(t) + y(t) = 8 \cos t, y(0) = 1, y'(0) = -1. \quad 7$$

6. a) Find the Fourier sine transform of $e^{-|x|}$. Hence show

$$\text{that } \int_0^{\infty} \frac{x \sin mx}{1+x^2} \, dx = \frac{\pi e^{-m}}{2}, m > 0. \quad 7$$

- b) Solve the equation :

$$\left(D^2 + \frac{1}{x} D\right)y = \frac{12 \log x}{x^2} \quad 7$$

7. a) Solve the equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$, given that

$$u(0, t) = u(l, t) = 0, u(x, 0) = f(x) \text{ and}$$

$$\frac{\partial u}{\partial t}(x, 0) = 0, 0 < x < l. \quad 7$$

- b) Six coins are tossed 6400 times. Using Poisson distribution find the approximate probability of getting six heads 8 times. 7

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