Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS/M.TECH (CSE)/SEM-2/CSEM-201/2010

2010

ADVANCED MATHEMATICS

Time Allotted : 3 Hours Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

1. Answer any *five* questions. 5 × 2 = 10

   i) Find the Laplace Transform of the function $(5e^{2t} - 3)^2$.

   ii) Find the general solution of the equation $\frac{d^2y}{dx^2} = e^{-2x}$.

   iii) Find the characteristic equation of the matrix

$$\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$

   iv) If $\alpha = (3, 6, 1)$ and $\beta = (2, 1, 2)$, then compute

   $3\alpha - 2\beta$.

v) For the following bi-variate data :

x :	– 2	– 1	0	1	2
y :	4	1	0	1	4

find correlation coefficient.

vi) Comment on the statement "a binomial variate has mean 4 and s.d. 3".

GROUP – B

Answer any *three* questions. $3 \times 5 = 15$

2. Verify Cayley-Hamilton theorem for the matrix

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{bmatrix}$$

. Hence find A^{-1} .

3. Solve the equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = e^x \sin x$.

4. Use convolution theorem to solve $L^{-1}\left\{\frac{1}{(s-1)(s-2)}\right\}$.

5. Let (x, y) and (u, v) represent two sets of bivariate data such that $u = ax + b$ and $v = cy + d$ then prove that

$$r_{uv} = \frac{a\ c}{|a|\ |c|}\ r_{xy}$$

where a, b, c, d are constants.

6. Find the mean and s.d. of the random variable whose p.d.f. is $f(x) = \frac{1}{2}\ e^{-|x|}$, $-\infty < x < \infty$.

GROUP – C

Answer any *three* questions. $3 \times 15 = 45$

7. a) Evaluate $\int_0^\infty e^{-3t}\, t \sin t \, dt$.

   b) Evaluate $L^{-1}\left\{\frac{2s^2 - 4}{(s-3)(s^2 - s - 2)}\right\}$.

   c) Solve the differential equation using Laplace Transform

   $\frac{d^2y}{dt^2} + y = \sin 2t,\ y(0) = 0,\ \frac{dy}{dt} = 1$ at $t = 0$.

   5 + 5 + 5

8. a) Find the eigenvalues and eigenvectors of the matrix :

   $$A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}.$$

   b) Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 1 = xe^x \sin x$. 8 + 7

9. a) Find the Laplace Transform of the function

   $f(t) = 0,\ 0 < t \le 1$

   $= t,\ 1 < t \le 2$

   $= 0,\ t > 2.$

   b) Identify which one of the following is regression equation of y on x and x on y :

   $2x + 3y = 10$ and $x + 6y = 6$.

   Find mean of x and y and the correlation coefficient of x, y.

   Estimate x when $y = -1$. 7 + 8

10. a) Find mean and s.d. of the Binomial Distribution.

b) The length of bolts produced by a machine is normally distributed with mean 4 and s.d. 0·5. A bolt is defective if its length does not lie in the interval (3·8, 4·3). Find the percentage of defective bolts produced by the machine.

$$\left[\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0\cdot6} e^{-\frac{t^2}{2}} dt = 0\cdot7257 \ , \quad \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0\cdot4} e^{-\frac{t^2}{2}} dt = 0\cdot6554 \right]$$

c) What is the chance that a leap year selected at random will contain 53 wednesdays ? 5 + 7 + 3

11. a) In a certain class 25% of the students failed in Mathematics, 15% faled in Chemistry, 10% failed in both Mathematics and Chemistry. A student is selected at random.

i) If he failed in Mathematics, what is the probability that he failed in Chemistry ?

ii) If he failed in Chemistry, what is the probability that he failed in Mathematics ?

iii) What is the probability that he failed in Mathematics & Chemistry ?

iv) What is the probability that he failed in Mathematics or Chemistry ?

b) A random variable X has the following probability distribution :

$X = x_i$: 0	1	2	3	4	5	6	7	8
f_i : k	$3k$	$5k$	$7k$	$9k$	$11k$	$13k$	$15k$	$17k$

i) Determine the value of k.

ii) Find $P (X < 3)$, $P (X \geq 3)$, $P (2 \leq X < 5)$

iii) What is the smallest value of x for which $P (X \leq x) > 0\cdot5$. 8 + 7