



Name : .....

Roll No. : .....

Invigilator's Signature : .....

**CS/M.TECH (CSE)/SEM-2/CSEM-201/2010  
2010**

**ADVANCED MATHEMATICS**

Time Allotted : 3 Hours

Full Marks : 70

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

**GROUP – A**

1. Answer any *five* questions. 5 × 2 = 10

i) Find the Laplace Transform of the function  $(5e^{2t} - 3)^2$ .

ii) Find the general solution of the equation  $\frac{d^2y}{dx^2} = e^{-2x}$ .

iii) Find the characteristic equation of the matrix

$$\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$

iv) If  $\alpha = (3, 6, 1)$  and  $\beta = (2, 1, 2)$ , then compute

$$3\alpha - 2\beta.$$



v) For the following bi-variate data :

$x :$	- 2	- 1	0	1	2
$y :$	4	1	0	1	4

find correlation coefficient.

vi) Comment on the statement “a binomial variate has mean 4 and s.d. 3”.

### GROUP – B

Answer any *three* questions.

3 × 5 = 15

2. Verify Cayley-Hamilton theorem for the matrix

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{bmatrix} . \text{ Hence find } A^{-1} .$$

3. Solve the equation  $\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} = e^x \sin x$ .

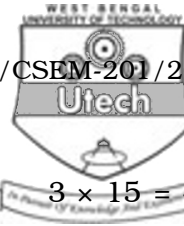
4. Use convolution theorem to solve  $L^{-1} \left\{ \frac{1}{(s-1)(s-2)} \right\}$ .

5. Let  $(x, y)$  and  $(u, v)$  represent two sets of bivariate data such that  $u = ax + b$  and  $v = cy + d$  then prove that

$$r_{uv} = \frac{a}{|a|} \frac{c}{|c|} r_{xy}$$

where  $a, b, c, d$  are constants.

6. Find the mean and s.d. of the random variable whose p.d.f. is  $f(x) = \frac{1}{2} e^{-|x|}$ ,  $-\infty < x < \infty$ .

**GROUP – C**Answer any *three* questions.

3 × 15 = 45

7. a) Evaluate  $\int_0^{\infty} e^{-3t} t \sin t \, dt$ .

b) Evaluate  $L^{-1} \left\{ \frac{2s^2 - 4}{(s - 3)(s^2 - s - 2)} \right\}$ .

c) Solve the differential equation using Laplace Transform

$$\frac{d^2 y}{dt^2} + y = \sin 2t, \quad y(0) = 0, \quad \frac{dy}{dt} = 1 \text{ at } t = 0.$$

5 + 5 + 5

8. a) Find the eigenvalues and eigenvectors of the matrix :

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}.$$

b) Solve  $\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + 1 = x e^x \sin x$ . 8 + 7

9. a) Find the Laplace Transform of the function

$$\begin{aligned} f(t) &= 0, \quad 0 < t \leq 1 \\ &= t, \quad 1 < t \leq 2 \\ &= 0, \quad t > 2. \end{aligned}$$

b) Identify which one of the following is regression equation of  $y$  on  $x$  and  $x$  on  $y$  :

$$2x + 3y = 10 \text{ and } x + 6y = 6.$$

Find mean of  $x$  and  $y$  and the correlation coefficient of  $x, y$ .

Estimate  $x$  when  $y = -1$ .

7 + 8



10. a) Find mean and s.d. of the Binomial Distribution.
- b) The length of bolts produced by a machine is normally distributed with mean 4 and s.d. 0.5. A bolt is defective if its length does not lie in the interval ( 3.8, 4.3 ). Find the percentage of defective bolts produced by the machine.

$$\left[ \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0.6} e^{-\frac{t^2}{2}} dt = 0.7257, \quad \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0.4} e^{-\frac{t^2}{2}} dt = 0.6554 \right]$$

- c) What is the chance that a leap year selected at random will contain 53 wednesdays ? 5 + 7 + 3
11. a) In a certain class 25% of the students failed in Mathematics, 15% failed in Chemistry, 10% failed in both Mathematics and Chemistry. A student is selected at random.
- If he failed in Mathematics, what is the probability that he failed in Chemistry ?
  - If he failed in Chemistry, what is the probability that he failed in Mathematics ?
  - What is the probability that he failed in Mathematics & Chemistry ?
  - What is the probability that he failed in Mathematics or Chemistry ?
- b) A random variable  $X$  has the following probability distribution :

$X = x_i : 0$	1	2	3	4	5	6	7	8
$f_i : k$	3k	5k	7k	9k	11k	13k	15k	17k

- Determine the value of  $k$ .
- Find  $P ( X < 3 )$ ,  $P ( X \geq 3 )$ ,  $P ( 2 \leq X < 5 )$
- What is the smallest value of  $x$  for which

$$P ( X \leq x ) > 0.5.$$

8 + 7