*Name* : .....

*Roll No.* : .....

*Invigilator's Signature* : .....

**CS/M.Tech (CSE)/SEM-1/CST-1103-A2/2009-10**

**2009**

# LOGIC & LOGIC PROGRAMMING

*Time Allotted* : 3 Hours Full Marks : 70

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

## GROUP – A
## ( Objective Type Questions )

1. State *True* or *False* for the following : 10 × 1 = 10

   i) A sentential symbol can take value 0 or 1.

   ii) Tautologies are those which are satisfiable under majority of value assignments.

   iii) Compactness theorem tells that if one finite subset is satisfiable then the whole set of wffs are satisfiable.

   iv) $\wedge$ & $\rightarrow$ form a complete set, *i.e* any Boolean function can be realized by them.

v) 'All subsets of natural numbers have a least element,' can be represented by sentential logic wff.

vi) De Morgan's law is not applicable in first order logic.

vii) Universal quantifier cannot be converted to an existential quantifier.

viii) A function in first order logic can take value 0 or 1.

ix) Completeness theorem tells 'any consistent set of formulae is satisfiable'.

x) Resolution is a proof technique that uses induction principle.

## GROUP – B
## ( Short Answer Type Questions )

Answer any *five* of the following. 5 × 3 = 15

2. Determine whether the following two are tautologies :

i) $(((P \rightarrow Q) \rightarrow P) \rightarrow P)$

ii) $(P \rightarrow Q) \vee (Q \rightarrow P)$

3. Convert the following sentences to first order logic wff :

   i) There is no set of which every set is a member

   ii) Any non-zero natural number is a successor of some number

   iii) $2^2 + 1^2 = 5$.

4. Explain what is a model. Show that the following wff cannot have natural numbers as its model :

   $\forall x \forall y \exists z \forall t (t \in z \leftrightarrow t = x \vee t = y)$.

5. Find a model of the wffs :

   i) $\exists x \forall y \neg y Ex$ where $E$ is a binary relation in the set of pairs $E = \{ (a, a), (b, a), (b, c), (c, c) \}$

   ii) $\exists x \forall y \neg y \in x$.

6. Prove 'Reductio Ad Absurdum,' *i.e* if $\Gamma; \varphi$ is inconsistent then $\Gamma \vdash \blacksquare \varphi$.

7. Write a Prolog program to find maximum of two numbers and explain with data how it works.

**GROUP – C**

**( Long Answer Type Questions )**

Answer any *three* of the following. 3 × 15 = 45

8. a) Let $G$ be an $n$-place Boolean function ( $n \geq 1$ ). Prove that we can find a wff $\alpha$ such that $G = B_\alpha^n$, that realizes the function $G$. 6

b) Say that a formula $\alpha$ is conjunctive normal form ( CNF ) iff it is a conjunction $\alpha = \gamma_l \wedge \ldots \wedge \gamma_k$, where each $\gamma_i$ is a disjunction $\gamma_i = \beta_{il} \vee \ldots \vee \beta_{in}$, and each $\beta_{ij}$ is either a sentence symbol or negation of a sentence symbol.

Find a formula in CNF that is tautologically equivalent to $A \leftrightarrow B \leftrightarrow C$. 4

c) Show that the following hold :

i) $\Sigma; \alpha \models \beta$ iff $\Sigma \models \alpha \rightarrow \beta$

ii) $\alpha \models \dashv \beta$ iff $\models \alpha \leftrightarrow \beta$

iii) If either $\Sigma \models \alpha$ or $\Sigma \models \beta$ then $\Sigma \models \alpha \vee \beta$. 5

9. a) Prove the 'Unique Readability Theorem' for sentential logic, that states : The five formula building operations when restricted to set of wffs,

   i) have ranges that are disjoint from each other and from sentence symbols,

   ii) are one-to-one. 7

   b) i) Explain the inference rule modus ponens.

   ii) Define what is a deduction of $\varphi$ from a set of formulae $\Gamma$.

   iii) Write down the forms of logical axioms. 4

   c) Prove that 'Generalization Theorem' *i.e* if $\Gamma \vdash \varphi$ and $x$ does not occur free in any formula in $\Gamma$, then $\vdash \forall x \varphi$. 4

10. a) Let $B$ be an initial set that is a subset of $U$. Let $F$ be a class of functions containing just two members $f$ & $g$, where $f : U \times U \to U$ & $g : U \to U$.

    Then prove the 'induction principle' *i.e.* if $C$ be generated from $B$ by the functions of $F$, and if $S$ is a subset of $C$ that includes $B$ and is closed under the functions of $F$, then $S = C$. 7

b) Show that the formula $x = y \rightarrow Pzfx \rightarrow Pzfy$ ( where $f$ is a one-place functional symbol and $P$ is a two place predicate symbol ) is valid. 4

c) Consider the following logical statements :

$$\forall\, xpet(x) \wedge small(x) \rightarrow appartmentpet(x),$$

$$\forall\, x\, cat(x) \vee dog(x) \rightarrow pet(x), \forall\, x\, poodle(x) \rightarrow$$

$$dog(x) \wedge small(x), poodle(fluffy)$$

Convert the above statements into Horn clauses and Prolog statements. Show how you can answer step-by-step. What is an apartmentpet ? 4

11. a) Find the clausal form of the following wff : 4

$$\forall\, x(\neg P(x,0) \rightarrow (\exists y(P(y, g(x)) \wedge \forall\, z(P(z), g(x)) \rightarrow P(y,z))))$$

b) What is unification ? Explain with example. Explain resolution principle both in cases of sentential logic and first order logic. Explain with example how resolution principle can be used for theorem proving in sentential logic. 6

c) State soundness theorem for first order logic. Assuming that all logical axioms are valid, prove the soundness theorem. Show that the following ( a logical axiom of Group 3 ) is valid :

$\left[\forall x(\alpha \to \beta) \to (\forall x \alpha \to \forall x \beta)\right]$. 5