Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/M.Tech(CSE)/SEM-1/CSEM-101/2012-13

2012

DISCRETE STRUCTURES

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Objective Type Questions)

1. Answer any *five* of the following : $5 \times 2 = 10$

i) Let $f : R \to R$ be defined by $f(x) = x^2 + 3$, then verify surjectivity.

ii) Prove that if a is the generator of a cyclic group $(G, \circ)$, then a^{-1} is generator of G.

iii) Examine whether the following permutation is even or odd :

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 4 & 3 & 2 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 1 & 4 & 7 & 2 & 5 & 8 & 6 \end{pmatrix}$$

iv) Find the number of pendant and internal vertices in a binary tree with 7 vertices.

v) Prove that there exists no graph with four edges having vertices of degree 4, 3, 2, 1.

vi) Prove that in a Ring (R, +, .) $a \cdot 0 = 0 \cdot a\, 0$ for all a in R.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Let R be a relation defined on Z by $'a\ R\ b'$ if and only if $(a-b)$ is divisible by 5 for $a, b \in Z$. Show that R is an equivalence relation.

3. Let S be set of all real matrices $\left\{\begin{pmatrix} a & b \\ -b & a \end{pmatrix} : a^2 + b^2 = 1\right\}$. Show that S forms a commutative group under matrix multiplication.

4. Let H be a subgroup of a group G. Then H is normal in G if and only if $xhx^{-1} \in H\ \forall h \in H$ and $\forall x \in G$.

5. If X is a Poisson variate with parameter θ then find (i) mean of X, (ii) variance of X.

6. Prove that a graph is a tree if and only if it is minimally connected.

GROUP – C
(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Prove that the order of each subgroup of a finite group is divisor of the order of the group.

   b) If $(G, \circ)$ be a finite cycle group generated by a, then prove that $O(G) = n$ if and only if $O(a) = n$.

   c) Prove that intersection of any two subgroups of a group $(G, \circ)$ is a subgroup of G. 5 + 5 + 5

8. a) In a bolt factory, machines *A, B* and *C* manufacture respectively 25%, 35% and 40% of the total of their output. 5%, 4% and 2% are defective bolts. A bolt is drawn at random from the product and is found to be defective bolt. What is the probability that it was manufactured by machines *A* and *B* ?

   b) The *pdf* of a continuous random variable *X* is given by

   $$f(x) = kx + \frac{1}{2}, \quad -1 < x < 1$$
   $$= 0, \qquad \text{elsewhere}$$

   Find (i) k and (ii) variance of the random variable *X*.

   c) In a certain factory turning razor blades, there is a small chance, 1/500 for any blade to be defective. The blades are in packets of 10. Calculate the approximate number of packets containing (i) no defective, (ii) two defective blades respectively in one consignment of 10000 packets. 5 + 5 + 5

9. a) Assuming that the set E of all real numbers of the form $a + b\sqrt{2}$ with a, b as integers forms a ring w.r.t. the ordinary addition and multiplication. Show that E is an integral domain. Is it a field ?

b) Using generating function solve the recurrence relation

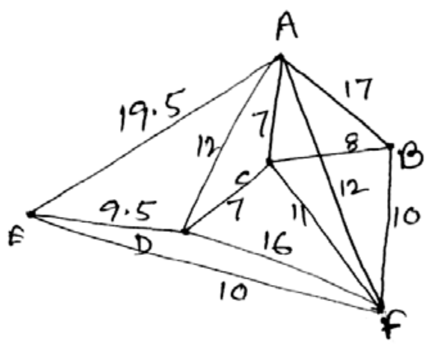
$$a_{n+2} - 5a_{n+1} + 6a_n = 2, \qquad a_0 = 1, a_1 = 2$$

c) Show that $A \times (B - C) = (A \times B) - (A \times C')$ where C' is the complement of C in U where U is a universal set.

5 + 5 + 5

10. a) Define tree. Prove that a tree with n number of vertices has $(n-1)$ edges.

b) Using Prim's Algorithm find the minimal spanning tree of the following graph :



7 + 8

=============