*Name* : ..............................................................

*Roll No.* : ..........................................................

*Invigilator's Signature* : ..............................................

**CS/M. Tech (CSE)/SEM-1/CSEM-102/2009-10**

## 2009

# DESIGN AND ANALYSIS OF ALGORITHM

*Time Allotted* : 3 Hours Full Marks : 70

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

**GROUP – A**

**( Multiple Choice Type Questions )**

1. Choose the correct alternatives of the following : $10 \times 1 = 10$

   i) Consider that a max-heap is represented using array. Which of the following is a valid representation of a max-heap ?

      a) 100, 25, 50, 10, 40, 5, 20

      b) 100, 50, 25, 10, 40, 5, 20

      c) 100, 25, 50, 5, 40, 10, 20

      d) none of these.

ii) The minimum worse case time complexity for any comparison based searching algorithm is

a) O ( $n$ log $n$ )
b) O ( $n$ )
c) O ( log $n$ )
d) O ( 1 ).

iii) Which data structure is used for breadth first traversal of a graph ?

a) stack
b) queue
c) heap
d) AVL-tree.

iv) How many times fib (3) is called during invocation of fib (6) with the following definition of fib ( ) ?

fib ( $n$ ) = fib ( $n$ – 1 ) + fib ( $n$ – 2 ) if $n$ > 2

fib ( 1 ) = fib ( 2 ) = 1

a) 3
b) 4
c) 5
d) 6.

v) Which of the following algorithm can be used to find single source shortest path of a graph containing negative weight cycles ?

a) Bellman-Ford's
b) Dijkstra's
c) Floyd's
d) Prim's.

vi) The time complexity of the following recurrence relation is $T(n) = 4^{*} T(n/2) + n$

a) $\Theta(n \log n)$  b) $\Theta(n^2)$

c) $\Theta(n^4)$  d) none of these.

vii) Which one of the following Boolean expression is in CNF ?

a) $X1 \cap (X2 U X3) U X4$  b) $X1 U (X2 \cap X3) U X4$

c) $X1 \cap (X2 \cap X3) U X4$  d) $X1 \cap (X2 U X3) \cap X4$.

viii) For a completely connected graph with $n$ vertices, the number of Hamiltonian circuit is

a) $n!$  b) $(n-1)!$

c) $\frac{1}{2} * (n-1)!$  d) $n^2$.

ix) The time complexity of Kruskal's algorithm to find minimum cost spanning tree of a graph of $n$ vertices and edges is

a) $O(n^2)$  b) $O(e \log e)$

c) $O(n \log n)$  d) $O(e^2)$.

x) The time complexity to delete the maximum element from a max-heap is

a) $O(n)$  b) $O(n \log n)$

c) $O(1)$  d) $O(\log n)$.

**GROUP – B**

**( Short Answer Type Questions )**

Answer any *three* of the following. $3 \times 5 = 15$

2. a) Define Big Oh ( O ) notation. 1

   b) If $f_1(n)$ is in $O(g_1(n))$ and $f_2(n)$ is in $O(g_2(n))$, then prove that $f_1(n) + f_2(n)$ is in $O(\max(g_1(n), g_2(n))$. 4

3. Consider the following chain of matrices of different dimensions as mentioned below :

| Matrix | Dimension |
|---|---|
| $A_1$ | $30 \times 35$ |
| $A_2$ | $35 \times 15$ |
| $A_3$ | $15 \times 5$ |
| $A_4$ | $5 \times 10$ |

Find out an optimal parenthization to multiply the above chain of matrices using dynamic programming. 5

4. Draw the recursion tree generated by the Quick Sort algorithm to sort the following list considering 1st element of the list as the pivot element.

   11, 6, 4, 10, 16, 8, 1, 3, 15, 12, 5, 7, 14, 13, 9, 2 5

5. Prove that for any comparison based sorting algorithm the minimum worst case time complexity is O ( $n \log n$ ). 5

6. Consider that there are three pegs $X$, $Y$ and $Z$ and peg $X$ contains some finite number of disks of different diameters. Also consider that the disks are placed in decreasing order of diameter at peg $X$ that is the smallest diameter disk is at the top of peg $X$ and the largest diameter disk is at the bottom of the peg $X$. Now the task is to move all the disks from peg $X$ to peg $Z$ using peg $Y$ as a temporary peg subject to the conditions that only one disk can be move from top of any peg to the top of another peg at a time and a larger diameter disk can never be placed on a smaller diameter disk.

   a) Write a recursive algorithm to solve the problem. 3

   b) Draw the recursion tree generated by the above algorithm for 3 number of disks. 2

**GROUP – C**

**( Long Answer Type Questions )**

Answer any *three* of the following. $3 \times 15 = 45$

7. a) What is the basic difference between divide and conquer and dynamic programming ? Explain with example. 4

   b) Write Floyd's algorithm for all pairs shortest path of a directed graph. Find out the time complexity of your algorithm. 4 + 3

   c) Show the working of Floyd's algorithm on the following graph. 4

8. a) What is weighting rule for making the union of two disjoint sets ? 2

b) Using weighting rule write an algorithm to find the union of two disjoint sets. What is the complexity of your algorithm ? 3 + 2

c) Consider the universal set $U = \{ 1, 2, 3, 4, 5, 6, 7, 8 \}$ and initially PARENT (i) = – COUNT (i) = – 1; $1 \leq i \leq 8$ [ PARENT and COUNT are used in usual meaning ], show the working of the above algorithm for the following sequence of operations : 4

UNION ( 1, 2 ), UNION ( 3, 4 ), UNION ( 5, 6 ) UNION ( 7, 8 ), UNION ( 1, 3 ), UNION ( 5, 7 ) and UNION ( 1, 5 ).

d) Construct a max-heap with the following data :

100, 50, 25, 75, 5, 250, 500, 125 4

9. Consider that a set of $n$ jobs are to be processed in a single machine, associated with job $i$ is an integer deadline $D_i \geq 0$ and a profit $P_i \geq 0$. For any job $i$ the profit $P_i$ is earned if and only if the job is completed by its deadline. In order to complete a job one has to process the job on a machine for one unit of time. Only one machine is available for processing the jobs. A feasible solution for this problem is a subset $J$, of jobs such that each job in this subset can be completed by its deadline. The value of a feasible solution $J$ is the sum of the profits of the jobs in $J$. An optimal solution is a feasible solution with maximum value.

a) Write a Greedy Algorithm to find the optimal solution for the above problem. The algorithm should also produce the sequence of processing the jobs. 4

b) Find out the worst case time complexity of the above algorithm. 3

c) Consider that there are 4 jobs $J_1$, $J_2$, $J_3$ and $J_4$ with integer deadline 2, 3, 2 and 1 respectively and the profit associated with each job are 100, 10, 15 and 27 respectively. Find out the optimal solution using the above algorithm. 4

d) Find out the time complexity of Merge Sort. 4

10. a) Write an algorithm to find the minimum cost spanning tree of a graph using Kruskal's algorithm. 4

b) Show the working of your algorithm on the following graph. 4

c) Write DFS algorithm to traverse a graph. 4

d) Show the working of your algorithm on the following graph. 3

11. a) As you mean that Hamiltonian Circuit Problem [ HUM ] is NP-Complete, show that Travelling Salesman Problem [ TSP ] is also NP-Complete. 6

b) What do you understand by Travelling Salesman Problem with triangular inequality [ TSP – TI ] ? Present an approximation algorithm for TSP – TI that makes use of Prim's algorithm for minimum cost spanning tree ( description of Prim's algorithm is not required ). Show that the approximation algorithm has a ratio bound of 2. 2 + 3 + 4

12. Write short notes on any *three* of the following : $3 \times 5$

a) Asymptotic notation

b) Tail recursion

c) NP-Hard problem

d) Single Source Shortest Path Problem.