



Name :

Roll No. :

Invigilator's Signature :

CS/M.Tech(CSE)/SEM-1/MCSE-101/2012-13

2012

ADVANCED ENGINEERING MATHEMATICS

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following :
 $10 \times 1 = 10$

i) If $T : V \rightarrow W$ be a linear mapping, then for all
 $a, b, c \in F$ and $\alpha, \beta \in V$

a) $T(a\alpha - b\beta) = aT(\alpha) + bT(\beta)$

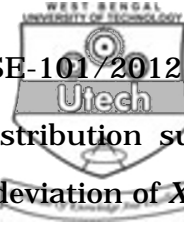
b) $T(a\alpha + b\beta) = aT(\alpha) + bT(\beta)$

c) $T(a\alpha - b\beta) = aT(\alpha) - bT(\beta)$

d) $T(a\alpha + b\beta) = aT(\alpha) - bT(\beta).$



- ii) The $\ker T$ of the linear mapping $T: V \rightarrow W$, is a
- a) subspace of W
 - b) subspace of $V \cap W$
 - c) subspace of V
 - d) none of these.
- iii) A linear mapping $T: V \rightarrow W$ is invertible if and only if T is
- a) one-to-one and onto
 - b) one-to-one and into
 - c) one-to-many
 - d) into.
- iv) If $-\lambda^3 + b\lambda^2 - 9\lambda + 4$ is the characteristic polynomial of a matrix A then $\det(A)$ is
- a) 2
 - b) -9
 - c) 6
 - d) 4.
- v) If 2, 4, 4, 4 are all the eigenvalues of a matrix, then the algebraic multiplicity of the eigenvalue 4 is
- a) 1
 - b) 2
 - c) 0
 - d) 3.
- vi) The z -transform of $\{1\}$ is
- a) $\frac{z}{z+1}$
 - b) $\frac{z}{z-1}$
 - c) $\frac{1}{z-1}$
 - d) $\frac{1}{z+1}$.



vii) A random variable X has a Poisson distribution such that $P(1) = P(2)$. Then the standard deviation of X is

- a) 0
- b) 2
- c) $\sqrt{2}$
- d) -2.

viii) $f(x)$ is a periodic function, if for $\lambda > 0$

- a) $f(x + \lambda) = f(x), \forall x$
- b) $f(x - \lambda) = f(x), \forall x$
- c) $f(x \pm \lambda) = f(x), \forall x$
- d) none of these.

ix) The period of the function $f(x) = \cos(2\pi x)$ is

- a) 0
- b) 1
- c) 2
- d) 5.

x) $f(x) = x^2, x \in [-1, 1]$. Its Fourier series contains

- a) only sine terms
- b) only cosine terms
- c) both (a) and (b)
- d) none of these.

xi) The period of $f(x) = \tan(x)$ is

- a) 0
- b) 1
- c) π
- d) $\sqrt{2}$.



xii) If $f(x) = x \sin(x)$, $x \in [-\pi, \pi]$ is expressed as Fourier Series then

- a) $a_0 = 2$ b) $a_0 = 0$
 c) $a_0 = 4$ d) $a_0 = 1$.

xiii) The interval of $f(t)$ defined for which Fourier transform is possible is

- a) $-\infty < t < 0$ b) $0 < t < \infty$
 c) $-\infty < t < \infty$ d) none of these.

xiv) If $F(s)$ is the Fourier transform of $f(t)$ then Fourier transform of $f(t) \cos(t)$ is

- a) $\frac{1}{2} F(s - a)$
 b) $\frac{1}{2} F(s + a)$
 c) $\frac{1}{2} [F(s + a) + F(s - a)]$
 d) none of these.

xv) Fourier transform of $f(t) = 1$, $0 < t < 1$ is

- a) $\frac{1}{s}$ b) $1 - \cos(s)$
 c) $s \{1 - \cos(s)\}$ d) $\frac{1 - \cos(s)}{s}$.

xvi) The Fourier transform of $f(x + a)$, $a \neq 0$ is

- a) $F(s)$ b) $e^{iax} F(s)$
- c) $e^{-iax} F(s)$ d) e^{iax} .

xvii) The kernel function of Fourier transform is

- a) e^{isx} b) e^{-isx}
- c) $\cos(sx)$ d) $\sin(sx)$.

xviii) $f(x) = x^3 \cos(x)$, $-1 \leq x \leq 1$ is

- a) an even function b) an odd function
c) neither even nor odd d) none of these.

GROUP – B

(Short Answer Type Questions)

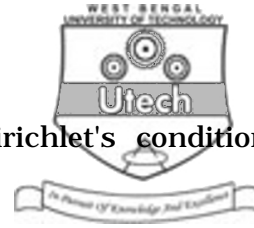
Answer any *three* of the following. $3 \times 5 = 15$

2. Find the linear mapping $T : R^3 \rightarrow R^2$ which maps the basis vectors $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$ of R^3 to the vectors $(1, 1)$, $(2, 3)$, $(-1, 2)$ respectively. Find $T(1, 2, 0)$.
3. Find eigenvalues and eigenvectors of the matrix
$$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}.$$
4. Find $Z^{-1} \left(\frac{z+2}{z^2-5z+6} \right)$

OR

Find the Fourier series of $f(x) = x$, $|x| \leq 2$.

5. Calculate Fourier transform for the function $f(x) = x, |x| \leq a$.



6. Check whether the function satisfies Dirichlet's condition and find its fourier series :

$$f(x) = 0, -\pi < x \leq 0$$

$$= \frac{\pi x}{4}, 0 < x \leq \pi.$$

GROUP - C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. Find the linear transform $T : R^3 \rightarrow R^3$ if $T(1, 0, 0) = (2, 3, 4)$, $T(0, 1, 0) = (1, 2, 3)$, $T(0, 1, 0) = (1, 1, 1)$. Find the matrix of T relative to the ordered basis $(\alpha_1, \alpha_2, \alpha_3)$ where $\alpha_1 = (1, 0, 0)$, $\alpha_2 = (0, 1, 0)$, $\alpha_3 = (0, 0, 1)$. Deduce that T is not invertible. $5 + 5 + 5$

8. If $A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & -2 & -1 \\ 2 & 3 & 2 \end{bmatrix}$, prove by method of induction, $A^n - A^{n-2} = A^2 - I, \forall n \geq 3$. Hence find A^{100} and A^{-1} . $7 + 4 + 4$

9. a) Using z-transform solve $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$, given $y_0 = y_1 = 0$.
- b) Two random processes $X(t)$ and $Y(t)$ are defined as $X(t) = A \cos(\omega_0 t) + B \sin(\omega_0 t)$ and $Y(t) = B \cos(\omega_0 t) - A \sin(\omega_0 t)$. Show that $X(t)$ and $Y(t)$ are jointly wide sense stationary, if A and B are uncorrelated random variables with zero means and the same variance and ω_0 is a constant. $8 + 7$

OR



Find the Fourier series of $f(x)$, where

$$f(x) = e^{-x}, x \in [-1, 1]$$

Check Dirichlet's condition for $f(x)$.

10 + 5

10. Find the Fourier cosine series for $f(x) = x$, $0 \leq x \leq \pi$.

11. a) Find the inverse of Fourier cosine transform of

$$F(s) = \frac{1}{1 + s^2}.$$

b) Prove that

$$\Im \left\{ \frac{\cos(3x)}{x^2 + 2} \right\} = \frac{\pi}{2\sqrt{2}} \left[e^{-\sqrt{2}|s+3|} + e^{-\sqrt{2}|s-3|} \right]$$

OR

Using Laplace transform, evaluate $\int \frac{e^{-at} \sin^2 t}{t} dt$.

7 + 8

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