Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/M.Tech(CSE)/SEM-1/MCSE-101/2012-13

2012

ADVANCED ENGINEERING MATHEMATICS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) If $T : V \to W$ be a linear mapping, then for all $a, b, c \in F$ and $\alpha, \beta \in V$

a) $T(a\alpha - b\beta) = aT(\alpha) + bT(\beta)$

b) $T(a\alpha + b\beta) = aT(\alpha) + bT(\beta)$

c) $T(a\alpha - b\beta) = aT(\alpha) - bT(\beta)$

d) $T(a\alpha + b\beta) = aT(\alpha) - bT(\beta)$.

ii) The ker T of the linear mapping $T : V \rightarrow W$, is a

a) subspace of W

b) subspace of $V \cap W$

c) subspace of V

d) none of these.

iii) A linear mapping $T : V \rightarrow W$ is invertible if and only if T is

a) one-to-one and onto

b) one-to-one and into

c) one-to-many

d) into.

iv) If $-\lambda^3 + b\lambda^2 - 9\lambda + 4$ is the characteristic polynomial of a matrix A then det (A) is

a) 2

b) – 9

c) 6

d) 4.

v) If 2, 4, 4, 4 are all the eigenvalues of a matrix, then the algebraic multiplicity of the eigenvalue 4 is

a) 1

b) 2

c) 0

d) 3.

vi) The z-transform of { 1 } is

a) $\dfrac{z}{z+1}$

b) $\dfrac{z}{z-1}$

c) $\dfrac{1}{z-1}$

d) $\dfrac{1}{z+1}$.

vii) A random variable X has a Poisson distribution such that $P(1) = P(2)$. Then the standard deviation of X is

a) 0 b) 2

c) $\sqrt{2}$ d) -2.

viii) $f(x)$ is a periodic function, if for $\lambda > 0$

a) $f(x + \lambda) = f(x), \forall x$

b) $f(x - \lambda) = f(x), \forall x$

c) $f(x \pm \lambda) = f(x), \forall x$

d) none of these.

ix) The period of the function $f(x) = \cos(2\pi x)$ is

a) 0 b) 1

c) 2 d) 5.

x) $f(x) = x^2, x \in [-1, 1]$. Its Fourier series contains

a) only sine terms b) only cosine terms

c) both (a) and (b) d) none of these.

xi) The period of $f(x) = \tan(x)$ is

a) 0 b) 1

c) π d) $\sqrt{2}$.

xii) If $f(x) = x \sin(x)$, $x \in [-\pi, \pi]$ is expressed as Fourier Series then

a) $a_0 = 2$ b) $a_0 = 0$

c) $a_0 = 4$ d) $a_0 = 1$.

xiii) The interval of $f(t)$ defined for which Fourier transform is possible is

a) $-\infty < t < 0$ b) $0 < t < \infty$

c) $-\infty < t < \infty$ d) none of these.

xiv) If $F(s)$ is the Fourier transform of $f(t)$ then Fourier transform of $f(t)\cos(t)$ is

a) $\frac{1}{2} F(s-a)$

b) $\frac{1}{2} F(s+a)$

c) $\frac{1}{2}[F(s+a) + F(s-a)]$

d) none of these.

xv) Fourier transform of $f(t) = 1$, $0 < t < 1$ is

a) $\frac{1}{s}$ b) $1 - \cos(s)$

c) $s\{1 - \cos(s)\}$ d) $\frac{1 - \cos(s)}{s}$.

xvi) The Fourier transform of $f(x+a)$, $a \neq 0$ is

a) $F(s)$ b) $e^{iax} F(s)$

c) $e^{-iax} F(s)$ d) e^{iax}.

xvii) The kernel function of Fourier transform is

a) e^{isx} b) e^{-isx}

c) $\cos(sx)$ d) $\sin(sx)$.

xviii) $f(x) = x^3 \cos(x), -1 \leq x \leq 1$ is

a) an even function b) an odd function

c) neither even nor odd d) none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Find the linear mapping $T : R^3 \rightarrow R^2$ which maps the basis vectors $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$ of R^3 to the vectors $(1, 1)$, $(2, 3)$, $(-1, 2)$ respectively. Find $T(1, 2, 0)$.

3. Find eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$.

4. Find $Z^{-1}\left(\dfrac{z+2}{z^2-5z+6}\right)$

OR

Find the Fourier series of $f(x) = x, \ |x| \leq 2$.

5. Calculate Fourier transform for the function $f(x) = x, \ |x| \leq a$.

6. Check whether the function satisfies Dirichlet's condition and find its fourier series :

$$f(x) = 0, \; -\pi < x \le 0$$
$$= \frac{\pi x}{4}, \; 0 < x \le \pi .$$

GROUP – C
(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. Find the linear transform $T : R^3 \rightarrow R^3$ if $T(1, 0, 0) = (2, 3, 4)$, $T(0, 1, 0) = (1, 2, 3)$, $T(0, 1, 0) = (1, 1, 1)$. Find the matrix of T relative to the ordered basis $\left(\alpha_1, \alpha_2, \alpha_3\right)$ where $\alpha_1 = (1, 0, 0)$, $\alpha_2 = (0, 1, 0)$, $\alpha_3 = (0, 0, 1)$. Deduce that T is not invertible. 5 + 5 + 5

8. If $A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & -2 & -1 \\ 2 & 3 & 2 \end{bmatrix}$, prove by method of induction, $A^n - A^{n-2} = A^2 - I, \; \forall n \ge 3$. Hence find A^{100} and A^{-1}.

7 + 4 + 4

9. a) Using z-transform solve $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$, given $y_0 = y_1 = 0$.

b) Two random processes $X(t)$ and $Y(t)$ are defined as $X(t) = A\cos(\omega_0 t) + B\sin(\omega_0 t)$ and $Y(t) = B\cos(\omega_0 t) - A\sin(\omega_0 t)$. Show that $X(t)$ and $Y(t)$ are jointly wide sense stationary, if A and B are uncorrelated random variables with zero means and the same variance and ω_0 is a constant. 8 + 7

OR

Find the Fourier series of $f(x)$, where

$$f(x) = e^{-x}, \ x \in [-1, 1]$$

Check Dirichlet's condition for $f(x)$. 10 + 5

10. Find the Fourier cosine series for $f(x) = x$, $0 \le x \le \pi$.

11. a) Find the inverse of Fourier cosine transform of

$$F(s) = \frac{1}{1 + s^2}.$$

b) Prove that

$$\Im\left\{\frac{\cos(3x)}{x^2 + 2}\right\} = \frac{\pi}{2\sqrt{2}}\left[e^{-\sqrt{2}|s+3|} + e^{-\sqrt{2}|s-3|}\right]$$

OR

Using Laplace transform, evaluate $\displaystyle\int \frac{e^{-at}\sin^2 t}{t}\,dt$.

7 + 8