Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/M.TECH(CSE)/SEM-1/CSEM-101/2011-12

2011

ADVANCED ENGINEERING MATHEMATICS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for the following : $10 \times 1 = 10$

i) What is a fundamental difference between a group and a ring ?

ii) Let G be a group and $a \in G$. If $O(a) = 17$, then $O(a^8)$ is

a) 17 b) 16

c) 8 d) 5.

iii) The set of all residue class Z_6 contains

a) 6 elements b) 5 elements

c) 7 elements d) none of these.

iv) In the group [1, –1 , i , – i] under multiplication, order of – i is

a) 0 b) 2

c) 4 d) – 4.

v) Subgroup of a cyclic group is cyclic.

a) Yes b) No.

vi) Which of the following is a ring ?

a) Z under × and +

b) Z under + and ×

c) The set $\{ 1, w, w^2 \}$ under × and +

d) none of these.

vii) Let R_1 and R_2 be regular sets defined over alphabet $\sum$ then

a) $R_1 \cap R_2$ is not regular b) $R_1 \cup R_2$ is regular

c) $\sum \cap R_2$ is not regular d) R_2^* is not regular.

viii) The intersection of CFL and regular language

a) is always regular b) is always context-free

c) both (a) and (b) d) need not be regular.

ix) A grammar that produces more than one parse tree for same sentence is called

a) ambiguous b) unambiguous

c) regular d) none of these.

x) The vernacular language English, if considered a formal language, is a

a) regular language

b) context-free language

c) context-sensitive language

d) none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following 3 × 5 = 15

2. Show that the set $G = \{ 1, 2, 3, 4, 5, 6 \}$ forms a cyclic group under the operation modulo 7. Find all generators of this group.

3. Verify whether the set Z_5 of all residue classes modulo 5 form a group w.r.t. multiplication of residue classes.
4. Show that intersection of subrings is a subring.
5. $M = (\{q_1, q_2, q_3\}, \{0.1\}, \delta, q_1, \{q_3\})$ is a non-deterministic finite automata, where δ is given by

   $\delta(q_{1,0}) = \{q_2, q_3\}$ $\qquad$ $\delta(q_{1,1}) = \{q_1\}$

   $\delta(q_{2,0}) = \{q_1, q_2\}$ $\qquad$ $\delta(q_{2,1}) = \{\varphi\}$

   $\delta(q_{3,0}) = \{q_2\}$ $\qquad$ $\delta(q_{3,1}) = \{q_1, q_2\}$

   Convert this into its corresponding DFA.
6. Construct a finite automata accepting all strings over $\{0, 1\}$ ending in 010 or 0010.

GROUP – C
(Long Answer Type Questions)

Answer any *three* of the following. $\qquad 3 \times 15 = 45$

7. a) Define Boolean Function with example.

   b) Construct the truth table of the Boolean function

   $f(x, y, z) = (yz + xz')(xy' + z)'$

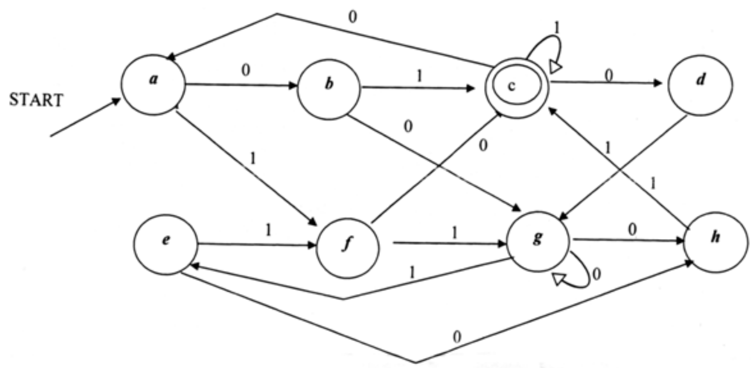
   c) A light in a room is to be controlled by 3 switches, located at three entrances. Design a simple series-parallel switching circuit such that flicking any one of the switches will change the state of the light.
8. a) Using the generating function solve the recurrence relation $a_n + 7a_{n-1} + 10a_{n-2} = 0, \;\forall\; n > 1$ and $a_0 = 3$ and $a_1 = 3$.

   b) Prove that every finite integral domain is fixed.
9. a) Prove that in a field F, the equations $a.x = b$ and $y.a = b$ have unique solutions where $a, b \in F$ and $a \neq 0$.

   b) Prove that the set P_n of all permutations of degree n on n symbols forms a group w.r.t. permutation multiplication.

10. a) What is distinguishable and indistinguishable states in finite automata ?

b) Use Myhill-Nerode theorem to minimize the following finite automata :



11. a) Using pumping lemma show that the set $L = \left\{ a^{i \wedge 2} \right\}$ is not regular.

b) Design a CFG for the language $L = \left\{ a^n b^m \mid n \neq m \right\}$.

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