Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS/M.Tech (CSE/SE)/SEM-1/PGCSE-104A/PGSE-104/2012-13

2012
ADVANCED ALGORITHMS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer Question No. 1 and any *four* from the rest.

1. Critically comment on the correctness for any four from the following statements :

   a) All NP Complete problems are NP-Hard as well, but the reverse is not true.

   b) Sequential search of a list is preferable to binary search under certain conditions.

   c) the performance of recursive algorithm for factorial calculation is better than the iterative counterpart.

   d) The greedy strategy gives optimal solution for 0/1 knapsack problem.

   e) Quick sort is a stable algorithm. $4 \times 3\frac{1}{2} = 14$

2. a) Show that an algorithm making at most a constant number of calls to polynomial time subroutines runs in polynomial time, but that a polynomial number of calls to polynomial time subroutines may result in an exponential algorithm

b) Prove that satisfiability of boolean formulas in 3-conjunctive normal form is NP-complete.

c) Define a clique. State the clique optimization problem. What will be the corresponding decision problem ?

d) State the condition under which a problem P 1 reduces to another problem P 2. 4 + 3 + 5 + 2

3. a) Define edge relaxation with example.

b) Let $G = (V, E)$ be a directed graph with weight function $W : E \rightarrow R$, and let $(u,v) \varepsilon E$. Then, immediately after relaxing edge (u, v), we have $d[v] < d[u] + w(u,v)$.

c) If s is the source vertex in the above graph, then prove that, if a sequence of relaxation steps sets Parent[s] to a non-NIL value, then G contains a negative weight cycle.

d) Analyse the complexity of Dijkstra's algorithm.

2 + 3 + 5 + 4

4. Ackermann's function isdefined as follows :

$A(0,n) = n + 1$ for $n >= 0$

$A(m,0) = A(m-1, 1)$ for $m > 0$

$A(m,n) = A(m-1, A(m, n-1))$ for $m > 0$ and $n > 0$

a) Write a recursive function to calculate Ackemann's function.

b) Calculate the following values :

$A(0, 0)$ $A(0, 9)$ $A(1, 8)$ $A(2, 2)$ $A(2, 3)$

c) Write a non-recursive function to calculate the above function. 4 + 5 + 5

5. a) State Master theorem.

   b) Give tight big-O bounds for T(n) for each of the following recurrence relations. In each case, state whether Master method can be applied or not.

   Assume $T(1) = 1$.

   a. $T(n) = 9T(n/3) + n$

   b. $T(n) = 2T(n/2) + n \log n$

   c. $T(n) = 2T(n/3) + 1$ $2 + 4 \times 3$

6. a) What is the largest possible number of internal nodes in a red-black tree with black height k ? What is the smallest possible number ?

   b) Does a red-black tree remains a Red black tree if its' root is changed from red to black ?

   c) Create a red-black tree by inserting nodes in the following order : 41, 38, 12, 19, 8.

   d) Show the successive deletions of the keys in the order 8, 12, 19, 31, 38, 41. $4 + 2 + 4 + 4$

7. a) Describe a θ (n lg n) time algorithm that, given a set S of n real numbers, and another real number x, determine whether or not there exist two elements in S whose sum is exactly x.

   b) Write an algorithm for Merge (A, p, q, r). Illustrate the operations of merge sort on the array A = (3, 41, 52, 26, 38, 57, 9, 49)

   c) Estimate the run-time complexity of your algorithm.

   $5 + 6 + 3$

8. a) Define three ways of representing a graph in computer memory.

   b) Differentiate between a strongly connected and a weakly connected graph with example.

   c) What is topological order ? For what kind of graphs is topological sorting defined ?

   d) Modify breadth first search procedure so that it will detect any (directed) cycles in the graph and indicate which vertices can not be placed in any topological order as they lie on a cycle.

   e) Estimate the run-time complexity of your algorithm.

3 + 2 + 2 + 4 + 3