Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/M.TECH(CSE)/SEM-1/PGCS(MCE)-101/2012-13

2012

ADVANCE ENGINEERING MATHEMATICS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer Q. No. 1 and *four* from the rest questions.

1. Answer the following questions : $7 \times 2 = 14$
   i) Let $f = u + iv$ be analytic in a domain *D*. Show that *f* is constant in *D* if $Arg\{f(z)\}$ is constant in *D*.
   ii) Define a simply connected domain and give an example of such domain.
   iii) Prove that $\Delta \log f(x) = \log\left\{1 + \dfrac{\Delta f(x)}{f(x)}\right\}$, where Δ is forward difference operator.
   iv) Determine the number of correct digits in the number *x*, given its relative error E_r ; $x = 0 \cdot 4785$, $E_r = 0 \cdot 2 \times 10^{-2}$.
   v) Define saddle point of a function of two variables.
   vi) If *A* & *B* be mutually independent event, then prove that $\overline{A}$ & $\overline{B}$ are also independent.
   vii) Prove that $P(a < X \le b) = F(b) - F(a)$, where *a* & *b* are real constants and *X* is a random variable.

2. a) Show that the following function is not differentiable at the origin;

$$f(x) = \frac{x^2 y^2 (y - ix)}{x^2 y^2 + (x - y)^2} \quad , z \neq 0$$

$$= 0 \text{ if } z = 0 .$$

b) Find the bilinear transformation which transforms the points $z = 1, -i, -1$ into the points $w = 0, i, \infty$ and show further that the area of the circle $|z| = 1$ is transformed into the half plane above the real axis in w-plane. 6 + 8

3. a) i) Explain the Newton-Raphson's method for computing a simple real root of an equation $f(x) = 0$.

ii) Give the geometrical significance of the method. When does the method fail ?

iii) Use this method to find a real root of the equation $3x - \cos x - 1 = 0$. 3 + (2 + 1) + 4

b) Solve the following system of equations :

$$2x_1 - 3x_2 + 4x_3 = 8$$

$$x_1 + x_2 + 4x_3 = 15$$

$$3x_1 + 4x_2 - x_3 = 8$$

by matrix factorization method. 4

4. a) State Cauchy's Residue theorem. Using this theorem to prove that $\int_0^{2\pi} \frac{\sin^2 \theta}{a + b\cos\theta} d\theta = \frac{2\pi}{b^2}\left[a^2 - \sqrt{a^2 - b^2}\right]; \; a > b > 0$.

b) Use Taylor's series method to compute y (0·25) correct to four decimal places if y (x) satisfies the equation $\frac{dy}{dx} = x^2 + y^2, \; y(0) = 0$. 7 + 7

5. a) Find the maxima & minima of the function $x^3 + y^3 - 3x - 12y + 20$. Find also the saddle points if exists.

b) State and prove Euler's theorem of several variable (1st order). Use it to prove that if $\phi(x, y) = \tan^{-1} \frac{x^2 + y^2}{x + y}$

then $x \frac{\partial \phi}{\partial x} + y \frac{\partial \phi}{\partial y} = \frac{1}{2} \sin 2\phi$. 5 + (2 + 4) + 3

6. a) There are three identical boxes, each provides with two drawers. In the first, each drawer contains a gold coin, in the third, each drawer contains a silver coin & in the second, one drawer contains a gold and the other a silver coin. A box is selected at random and one of the drawers is opened. If a gold coin is found, what is the probability that the box chosen is the second one ?

b) Show that the function $f(x)$ given by

$f(x) = k(x-9)(10-x)$, $9 \le x \le 10$

= 0 , elsewhere

is a *pdf* for a suitable value of the constant *k*. Find mean & variance of the distribution.

c) If X be any random variable with zero mean and unit variance, find the expectation of X^2. 5 + (2 + 2 + 2) + 3

7. a) State the following definitions :

i) Random variable

ii) Distribution function

iii) Axiomatic definition of probability

iv) Binomial distribution with parameters *n, p*.

b) Show that the probability of occurrence of only one of the events A and B is

$P(A) + P(B) - 2P(AB)$

c) A random variable X has the following probability distribution :

$X = x_i$:	0	1	2	3	4	5	6	7	8
f_i :	K	$3K$	$5K$	$7K$	$9K$	$11K$	$13K$	$15K$	$17K$

i) Determine the value of K

ii) Find $P(X < 3), P(X \geq 3), P(2 \leq X < 5)$

iii) What is the smallest value of x for which $P(X \leq x) > 0 \cdot 5$?

(1 + 1 + 2 + 1) + 3 + (1 + 3 + 2)

8. a) The regression lines for a bivariate sample are given by $x + 2y + 9 = 0$ and $2x + 5y + 7 = 0$ and let $S_x^2 = 12$. Calculate the value of $\overline{x}, \overline{y}, S_y$ and r.

b) Calculate the correlation coefficient and determine the regression line of y on x and x on y for the sample :

x :	7	5	4	10	3
y :	2	3	9	11	6

8 + 6

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