



Name : .....

Roll No. : .....

Invigilator's Signature : .....

**CS/M.TECH(CSE)/SEM-1/PGCS(MCE)-101/2012-13**

**2012**

**ADVANCE ENGINEERING MATHEMATICS**

Time Allotted : 3 Hours

Full Marks : 70

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words  
as far as practicable.*

Answer Q. No. 1 and four from the rest questions.

1. Answer the following questions :  $7 \times 2 = 14$
- i) Let  $f = u + iv$  be analytic in a domain  $D$ . Show that  $f$  is constant in  $D$  if  $\text{Arg}\{f(z)\}$  is constant in  $D$ .
  - ii) Define a simply connected domain and give an example of such domain.
  - iii) Prove that  $\Delta \log f(x) = \log \left\{ 1 + \frac{\Delta f(x)}{f(x)} \right\}$ , where  $\Delta$  is forward difference operator.
  - iv) Determine the number of correct digits in the number  $x$ , given its relative error  $E_r$ ;  $x = 0.4785$ ,  $E_r = 0.2 \times 10^{-2}$ .
  - v) Define saddle point of a function of two variables.
  - vi) If  $A$  &  $B$  be mutually independent event, then prove that  $\bar{A}$  &  $\bar{B}$  are also independent.
  - vii) Prove that  $P(a < X \leq b) = F(b) - F(a)$ , where  $a$  &  $b$  are real constants and  $X$  is a random variable.



2. a) Show that the following function is not differentiable at the origin;

$$f(z) = \frac{x^2 y^2 (y - ix)}{x^2 y^2 + (x - y)^2}, z \neq 0$$

$$= 0 \text{ if } z = 0.$$

- b) Find the bilinear transformation which transforms the points  $z = 1, -i, -1$  into the points  $w = 0, i, \infty$  and show further that the area of the circle  $|z| = 1$  is transformed into the half plane above the real axis in  $w$ -plane. 6 + 8

3. a) i) Explain the Newton-Raphson's method for computing a simple real root of an equation  $f(x) = 0$ .

- ii) Give the geometrical significance of the method. When does the method fail?

- iii) Use this method to find a real root of the equation  $3x - \cos x - 1 = 0$ . 3 + (2 + 1) + 4

- b) Solve the following system of equations :

$$2x_1 - 3x_2 + 4x_3 = 8$$

$$x_1 + x_2 + 4x_3 = 15$$

$$3x_1 + 4x_2 - x_3 = 8$$

by matrix factorization method.

4

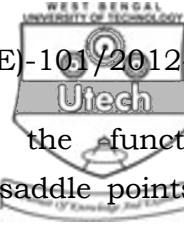
4. a) State Cauchy's Residue theorem. Using this theorem to

prove that  $\int_0^{2\pi} \frac{\sin^2 \theta}{a + b \cos \theta} d\theta = \frac{2\pi}{b^2} \left[ a^2 - \sqrt{a^2 - b^2} \right]; a > b > 0$ .

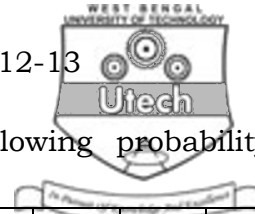
- b) Use Taylor's series method to compute  $y(0.25)$  correct to four decimal places if  $y(x)$  satisfies the equation

$$\frac{dy}{dx} = x^2 + y^2, y(0) = 0.$$

7 + 7



5. a) Find the maxima & minima of the function  $x^3 + y^3 - 3x - 12y + 20$ . Find also the saddle points if exists.
- b) State and prove Euler's theorem of several variable (1st order). Use it to prove that if  $\phi(x, y) = \tan^{-1} \frac{x^2 + y^2}{x + y}$  then  $x \frac{\partial \phi}{\partial x} + y \frac{\partial \phi}{\partial y} = \frac{1}{2} \sin 2\phi$ . 5 + ( 2 + 4 ) + 3
6. a) There are three identical boxes, each provides with two drawers. In the first, each drawer contains a gold coin, in the third, each drawer contains a silver coin & in the second, one drawer contains a gold and the other a silver coin. A box is selected at random and one of the drawers is opened. If a gold coin is found, what is the probability that the box chosen is the second one ?
- b) Show that the function  $f(x)$  given by  $f(x) = k(x - 9)(10 - x)$ ,  $9 \leq x \leq 10$    
  $= 0$ , elsewhere   
 is a *pdf* for a suitable value of the constant  $k$ . Find mean & variance of the distribution.
- c) If  $X$  be any random variable with zero mean and unit variance, find the expectation of  $X^2$ . 5 + ( 2 + 2 + 2 ) + 3
7. a) State the following definitions :
- Random variable
  - Distribution function
  - Axiomatic definition of probability
  - Binomial distribution with parameters  $n, p$ .
- b) Show that the probability of occurrence of only one of the events  $A$  and  $B$  is  $P(A) + P(B) - 2P(AB)$



- c) A random variable  $X$  has the following probability distribution :

$X = x_i :$	0	1	2	3	4	5	6	7	8
$f_i :$	$K$	$3K$	$5K$	$7K$	$9K$	$11K$	$13K$	$15K$	$17K$

- Determine the value of  $K$
- Find  $P(X < 3), P(X \geq 3), P(2 \leq X < 5)$
- What is the smallest value of  $x$  for which  $P(X \leq x) > 0.5$  ?

$$(1 + 1 + 2 + 1) + 3 + (1 + 3 + 2)$$

8. a) The regression lines for a bivariate sample are given by  $x + 2y + 9 = 0$  and  $2x + 5y + 7 = 0$  and let  $S_x^2 = 12$ . Calculate the value of  $\bar{x}, \bar{y}, S_y$  and  $r$ .

- b) Calculate the correlation coefficient and determine the regression line of  $y$  on  $x$  and  $x$  on  $y$  for the sample :

$x :$	7	5	4	10	3
$y :$	2	3	9	11	6

8 + 6

=====