CS/M.Tech (CI)/SEM-2/CI-2.3/09
DIGITAL CONTROL SYSTEM (SEMESTER - 2)

1. ....
Signature of Invigilator

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Signature of the Officer-in-Charge

Reg. No.

Roll No. of the Candidate

CS/M.Tech (CI)/SEM-2/CI-2.3/09

ENGINEERING & MANAGEMENT EXAMINATIONS, JULY – 2009

DIGITAL CONTROL SYSTEM (SEMESTER - 2)

Time : 3 Hours] [Full Marks : 70

INSTRUCTIONS TO THE CANDIDATES :

1. This Booklet is a Question-cum-Answer Booklet. The Booklet consists of **36 pages**. The questions of this concerned subject commence from Page No. 3.
2. You have to answer the questions in the space provided marked 'Answer Sheet'. Write on both sides of the paper.
3. **Fill in your Roll No. in the box** provided as in your Admit Card before answering the questions.
4. Read the instructions given inside carefully before answering.
5. You should not forget to write the corresponding question numbers while answering.
6. Do not write your name or put any special mark in the booklet that may disclose your identity, which will render you liable to disqualification. Any candidate found copying will be subject to Disciplinary Action under the relevant rules.
7. **Use of Mobile Phone and Programmable Calculator is totally prohibited in the examination hall.**
8. You should return the booklet to the invigilator at the end of the examination and should not take any page of this booklet with you outside the examination hall, **which will lead to disqualification**.
9. Rough work, if necessary is to be done in this booklet only and cross it through.

No additional sheets are to be used and no loose paper will be provided

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Marks Obtained

Question Number											***Total Marks***	***Examiner's Signature***
Marks Obtained												

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Head-Examiner/Co-Ordinator/Scrutineer

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DIGITAL CONTROL SYSTEM

SEMESTER - 2

Time : 3 Hours] [Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer Question No. 1 which is compulsory and any *four* from the rest.

5 ∞ 14 = 70

1. Answer the following questions briefly : 7 ∞ 2

i) The transfer function $\frac{M(Z)}{E(Z)}$ of the simulation diagram shown in Figure 1 is

Dia.

Fig. 1

a) $\dfrac{\overline{Z}^{1} - 1}{\overline{Z}^{1} + 1}$

b) $\dfrac{Z + 1}{Z - 1}$

c) $\dfrac{Z - 1}{Z + 1}$

d) $\dfrac{Z + \frac{1}{Z} - 1}{Z - \frac{1}{Z} + 1}$

Justify your answer.

ii) The signal flow graph in Figure 2 represents a system with system matrix A and output matrix C where

Dia.

Fig. 2

a) $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\frac{1}{4} & -\frac{1}{2} & -1 \end{bmatrix}$, $C = \begin{bmatrix} 1 \\ 1\cdot 11 \\ 0 \end{bmatrix}$

b) $A = \begin{bmatrix} 0 & -1 & -1\cdot 5 \\ 0 & 1 & 0 \\ 1\cdot 11 & 1 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 \\ 0 \\ 1\cdot 11 \end{bmatrix}$

c) $A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & -\frac{1}{2} & -\frac{1}{4} \end{bmatrix}$, $C = \begin{bmatrix} 1 \\ 1 \\ 1\cdot 11 \end{bmatrix}$

d) $A = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & \frac{1}{2} & \frac{1}{4} \end{bmatrix}$, $C = \begin{bmatrix} 1 \\ 1\cdot 11 \\ 1 \end{bmatrix}$

Choose the correct answer. $Y(Z)$ is output, $U(Z)$ is input. Justify your choice.

iii) Mapping of the points indicated in Figure 3 (points 1, 2, 3, 4 and 5) from S-plane to Z-plane with transformation $e^{ST} = Z$ will be as shown in

Dia.

Fig. 3

a) b)

c) d)

Justify your answer.

iv) Figure 4 shows a typical configuration of closed-loop discrete-time system. The output $C(Z)$ is determined by

Dia.

Fig. 4

a) $\dfrac{G_1 G_2(Z)R(Z)}{1+G_1(Z)G_2(Z)H(Z)}$ b) $\dfrac{G_1(Z)G_2 R(Z)}{1+G_1(Z)G_2 H(Z)}$

c) $\dfrac{G_1(Z)G_2(Z)R(Z)}{1+G_1(Z)G_2 H(Z)}$ d) $\dfrac{G_1 G_2(Z)R(Z)}{1+G_1 G_2(Z)H(Z)}$.

Select the correct answer. Justify.

v) Consider the state variable formulation

$$\underline{x}(k+1) = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \underline{x}(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} 3 & 1 \end{bmatrix} \underline{x}(k)$$

Output $y(2)$ is :

a) -1 b) 2

c) 1 d) -3.

Choose the correct value of output. Verify.

vi) Given $H(S) = \dfrac{1}{S+1}$, T = sampling period in sec.

If trapezoidal approximation is used for integration, then $H(Z)$ is

a) $\dfrac{T(1+Z^{-1})}{(1+T)-(1-T)Z^{-1}}$ b) $\dfrac{1+Z^{-1}}{(1+T)-(1-T)Z^{-1}}$

c) $\dfrac{T(1-Z)}{(1+T)-(1-T)Z}$ d) $\dfrac{\frac{T}{2}(1+Z^{-1})}{(1+\frac{T}{2})-(1-\frac{T}{2})Z^{-1}}$.

Select the correct answer with justification.

vii) If the S-plane poles of a 2nd order (underdamped) transfer function with damping ratio φ and natural frequency $w_n \frac{r}{s}$ result in Z-plane poles at $Z = r \angle \pm \theta$, then φ can be related to the Z-plane poles as

a) $\varphi = \frac{1}{T}\sqrt{l_n^2 r + \theta^2}$

b) $\varphi = \frac{l_n r}{l_n^2 r + \theta^2}$

c) $\varphi = -\frac{T}{l_n r}$

d) $\varphi = \frac{-l_n r}{\sqrt{l_n^2 r + \theta^2}}$

where T is the sampling period.

Choose the correct answer and justify.

2. a) An open-loop system characteristics is represented by :

$$G(Z) = \frac{K \prod^{m} (Z - Z_i)}{(Z-1)^N \prod^{p} (Z - Z_j)} \; ; \; Z_i \; \pi \; 1, Z_j \; \pi \; 1$$

and the value of N specifies the system type.

Derive expressions for steady-state errors in terms of error constants for unit step and unit ramp inputs for the unity feedback closed-loop system with sampling period T.

Hence, find the steady-state errors for unit step and unit ramp inputs for a unity feedback sampled data system having a sampling frequency of 20 Hz and an open-loop transfer function

$$G(S) = \frac{1 - e^{-Ts}}{S}\left[\frac{150}{S(S + 0{\cdot}7)}\right] .$$ 11

b) Determine the transfer function of the open-loop sampled data system shown in Figure 5. Assume that the samples are synchronised and $T = 0{\cdot}2$ sec. 3

Dia.

Fig. 5

3. Design a PI controller $D(Z)$ for a servo-system shown in Figure 6 which should meet the following performance specifications :

Dia.

Fig. 6

a) $Kv \geq 15$

b) Phase margin $\geq 60°$. 14

4. a) A closed loop discrete system has to be realised by state variable feedback utilising the position $x_1(k)$ and velocity $x_2(k)$ signal by pole assignment design with control input $u(k) = -K\underline{x}(k)$, where K is the gain matrix.

The desired closed-loop pole locations are $(0{\cdot}78 \pm j\,0{\cdot}40)$ and the discrete system state model is

$$\underline{x}(k+1) = \begin{bmatrix} 1 & 0{\cdot}07 \\ 0 & 1 \end{bmatrix} \underline{x}(k) + \begin{bmatrix} 0{\cdot}005 \\ 0{\cdot}06 \end{bmatrix} u(k) \text{ and}$$

$$y(k) = [\,1 \quad 0\,]\,\underline{x}(k).$$

Determine the gain matrix and show how the feedback gains may be implemented. 11

b) The state model of a linear time invariant discrete-time system is defined by :

$$\underline{x}(k+1) = A\underline{x}(k) + B\underline{u}(k)\ ;\ \underline{x}(kT) \triangleq \underline{x}(k).$$

$$\underline{y}(k) = C\underline{x}(k) + D\underline{u}(k).$$

Derive the expressions of $\underline{x}(k)$ and $\underline{y}(k)$ in terms of the state transition matrix. 3

5. a) The block diagram model of a temperature control system is shown in Figure 7. Determine the closed loop transfer function in Z-domain and then derive an expression for the output temperature $C(KT)$ at sampling instant when a unit step input is applied to the system. Sketch the nature of the output time response for first five output samples. 10

Dia.

Fig. 7

b) Show that the closed loop sampled data system with the first order plant $G(S) = \frac{A}{S}$ shown in Figure 8 is conditionally stable. 4

Dia.

Fig. 8

6. a) Apply Jury's test to determine the stability of a discrete time system whose closed loop characteristic equation is given by : 8

$$Z^3 + 3\cdot3Z^2 + 4Z + 0\cdot8 = 0$$

b) Determine the Inverse Z-transform $f(KT)$ for the function

$$G(Z) = \frac{-10(11Z^2 - 15Z + 6)}{(Z^2 - 2Z + 1)(Z - 2)}$$ 4

c) Solve the following difference equation :

$$y\,(K+2) + 5y\,(K-1) + 6y\,(K) = u\,(K)$$

for step input of $u\,(K) = 1$

and initial conditions are : $y\,(0) = 0$, $y\,(1) = 1$. 2

7. a) A discrete data system is given as

$$\underline{x}\,(K+1) = A\,\underline{x}\,(K) + Bu\,(K)$$

$$y\,(K) = C\,\underline{x}\,(K)$$

where $A = \begin{bmatrix} 0 & 1 \\ -15 & 8 \end{bmatrix}$; $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$; $C = [\ 1\ 0\ ]$

Evaluate the state transition matrix. 8

b) Consider that the digital process of the system shown in Figure 9 is described by the transfer function $D_2\,(Z)$ and consists of a digital controller $D_1\,(Z)$.

Dia.

Fig. 9

Design a controller transmittance so that the system will have dead beat response when the input is a unit step and the error between output and input is zero in the steady-state. 6

END