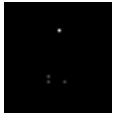
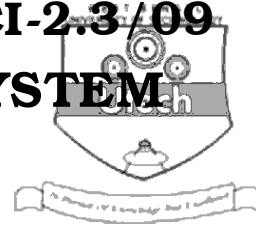


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CS / M.Tech (CI) / SEM-2 / CI-2.3 / 09

DIGITAL CONTROL SYSTEM**SEMESTER - 2**

Time : 3 Hours]

[Full Marks : 70

*The figures in the margin indicate full marks.**Candidates are required to give their answers in their own words as far as practicable.*Answer Question No. 1 which is compulsory and any *four* from the rest.

5 ∞ 14 = 70

1. Answer the following questions briefly :

7 ∞ 2

- i) The transfer function $\frac{M(Z)}{E(Z)}$ of the simulation diagram shown in Figure 1 is

Dia.

Fig. 1

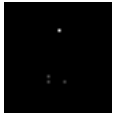
a) $\frac{\bar{Z}^1 - 1}{\bar{Z}^1 + 1}$

b) $\frac{Z + 1}{Z - 1}$

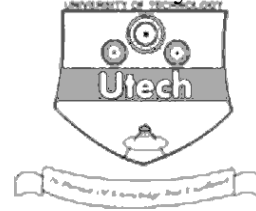
c) $\frac{Z - 1}{Z + 1}$

d) $\frac{Z + \frac{1}{Z} - 1}{Z - \frac{1}{Z} + 1}$

Justify your answer.



- ii) The signal flow graph in Figure 2 represents a system with system matrix A and output matrix C where



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Fig. 2

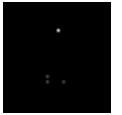
$$\text{a) } A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\frac{1}{4} & -\frac{1}{2} & -1 \end{bmatrix}, C = \begin{bmatrix} 1 \\ 1 \cdot 11 \\ 0 \end{bmatrix}$$

$$\text{b) } A = \begin{bmatrix} 0 & -1 & -1 \cdot 5 \\ 0 & 1 & 0 \\ 1 \cdot 11 & 1 & 0 \end{bmatrix}, C = \begin{bmatrix} 1 \\ 0 \\ 1 \cdot 11 \end{bmatrix}$$

$$\text{c) } A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & -\frac{1}{2} & -\frac{1}{4} \end{bmatrix}, C = \begin{bmatrix} 1 \\ 1 \\ 1 \cdot 11 \end{bmatrix}$$

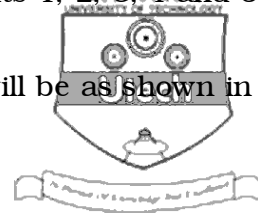
$$\text{d) } A = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & \frac{1}{2} & \frac{1}{4} \end{bmatrix}, C = \begin{bmatrix} 1 \\ 1 \cdot 11 \\ 1 \end{bmatrix}$$

Choose the correct answer. $Y(Z)$ is output, $U(Z)$ is input. Justify your choice.



iii) Mapping of the points indicated in Figure 3 (points 1, 2, 3, 4 and 5) from

S-plane to Z-plane with transformation $e^{ST} = Z$ will be as shown in



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Fig. 3

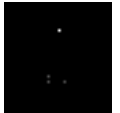
a)

b)

c)

d)

Justify your answer.



- iv) Figure 4 shows a typical configuration of closed-loop discrete-time system. The output $C(Z)$ is determined by



Dia.

Fig. 4

- a) $\frac{G_1 G_2(Z) R(Z)}{1 + G_1(Z) G_2(Z) H(Z)}$ b) $\frac{G_1(Z) G_2 R(Z)}{1 + G_1(Z) G_2 H(Z)}$
 c) $\frac{G_1(Z) G_2(Z) R(Z)}{1 + G_1(Z) G_2 H(Z)}$ d) $\frac{G_1 G_2(Z) R(Z)}{1 + G_1 G_2(Z) H(Z)}$.

Select the correct answer. Justify.

- v) Consider the state variable formulation

$$\underline{x}(k+1) = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \underline{x}(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k)$$

$$y(k) = [3 \ 1] \underline{x}(k)$$

Output $y(2)$ is :

- a) -1 b) 2
 c) 1 d) -3.

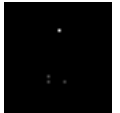
Choose the correct value of output. Verify.

- vi) Given $H(S) = \frac{1}{S+1}$, T = sampling period in sec.

If trapezoidal approximation is used for integration, then $H(Z)$ is

- a) $\frac{T(1+Z^{-1})}{(1+T)-(1-T)Z^{-1}}$ b) $\frac{1+Z^{-1}}{(1+T)-(1-T)Z^{-1}}$
 c) $\frac{T(1-Z)}{(1+T)-(1-T)Z}$ d) $\frac{\frac{T}{2}(1+Z^{-1})}{(1+\frac{T}{2})-(1-\frac{T}{2})Z^{-1}}$.

Select the correct answer with justification.



- vii) If the S-plane poles of a 2nd order (underdamped) transfer function with damping ratio φ and natural frequency ω_n result in Z-plane poles at $Z = r \angle \pm \theta$, then φ can be related to the Z-plane poles as

a) $\varphi = \frac{1}{T} \sqrt{l_n^2 r + \theta^2}$

b) $\varphi = \frac{l_n r}{l_n^2 r + \theta^2}$

c) $\varphi = -\frac{T}{l_n r}$

d) $\varphi = \frac{-l_n r}{\sqrt{l_n^2 r + \theta^2}}$

where T is the sampling period.

Choose the correct answer and justify.

2. a) An open-loop system characteristics is represented by :

$$G(Z) = \frac{K \prod_{i=1}^m (Z - Z_i)}{(Z - 1)^N \prod_{j=1}^p (Z - Z_j)} ; Z_i \neq 1, Z_j \neq 1$$

and the value of N specifies the system type.

Derive expressions for steady-state errors in terms of error constants for unit step and unit ramp inputs for the unity feedback closed-loop system with sampling period T .

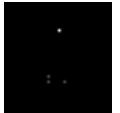
Hence, find the steady-state errors for unit step and unit ramp inputs for a unity feedback sampled data system having a sampling frequency of 20 Hz and an open-loop transfer function

$$G(S) = \frac{1 - e^{-Ts}}{S} \left[\frac{150}{S(S + 0.7)} \right] \quad 11$$

- b) Determine the transfer function of the open-loop sampled data system shown in Figure 5. Assume that the samples are synchronised and $T = 0.2$ sec. 3

Dia.

Fig. 5



3. Design a PI controller $D(Z)$ for a servo-system shown in Figure 6 which should meet the following performance specifications :



Dia.

Fig. 6

- a) $K_v \geq 15$
- b) Phase margin $\geq 60^\circ$. 14
4. a) A closed loop discrete system has to be realised by state variable feedback utilising the position $x_1(k)$ and velocity $x_2(k)$ signal by pole assignment design with control input $u(k) = -K\underline{x}(k)$, where K is the gain matrix.

The desired closed-loop pole locations are $(0.78 \pm j 0.40)$ and the discrete system state model is

$$\underline{x}(k+1) = \begin{bmatrix} 1 & 0.07 \\ 0 & 1 \end{bmatrix} \underline{x}(k) + \begin{bmatrix} 0.005 \\ 0.06 \end{bmatrix} u(k) \text{ and}$$

$$y(k) = [1 \ 0] \underline{x}(k).$$

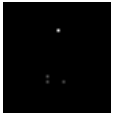
Determine the gain matrix and show how the feedback gains may be implemented. 11

- b) The state model of a linear time invariant discrete-time system is defined by :

$$\underline{x}(k+1) = A\underline{x}(k) + B\underline{u}(k); \underline{x}(kT) \triangleq \underline{x}(k).$$

$$\underline{y}(k) = C\underline{x}(k) + D\underline{u}(k).$$

Derive the expressions of $\underline{x}(k)$ and $\underline{y}(k)$ in terms of the state transition matrix. 3



5. a) The block diagram model of a temperature control system is shown in Figure 7. Determine the closed loop transfer function in Z-domain and then derive an expression for the output temperature C (KT) at sampling instant when a unit step input is applied to the system. Sketch the nature of the output time response for first five output samples.



10

Dia.

Fig. 7

- b) Show that the closed loop sampled data system with the first order plant $G(S) = \frac{A}{S}$ shown in Figure 8 is conditionally stable.

4

Dia.

Fig. 8

6. a) Apply Jury's test to determine the stability of a discrete time system whose closed loop characteristic equation is given by :

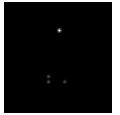
8

$$Z^3 + 3.3Z^2 + 4Z + 0.8 = 0$$

- b) Determine the Inverse Z-transform f (KT) for the function

$$G(Z) = \frac{-10(11Z^2 - 15Z + 6)}{(Z^2 - 2Z + 1)(Z - 2)}$$

4



- c) Solve the following difference equation :

$$y(K+2) + 5y(K-1) + 6y(K) = u(K)$$

for step input of $u(K) = 1$

and initial conditions are : $y(0) = 0, y(1) = 1$.



2

7. a) A discrete data system is given as

$$\underline{x}(K+1) = A\underline{x}(K) + Bu(K)$$

$$y(K) = C\underline{x}(K)$$

$$\text{where } A = \begin{bmatrix} 0 & 1 \\ -15 & 8 \end{bmatrix}; B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}; C = [1 \ 0]$$

Evaluate the state transition matrix.

8

- b) Consider that the digital process of the system shown in Figure 9 is described by the transfer function $D_2(Z)$ and consists of a digital controller $D_1(Z)$.

Dia.

Fig. 9

Design a controller transmittance so that the system will have dead beat response when the input is a unit step and the error between output and input is zero in the steady-state.

6

END