

# CS/M-TECH (CHE)/SEM-1/CHE-3/2010-11 <br> 2010-11 <br> ADVANCED FLUID MECHANICS 

Time Allotted: 3 Hours
Full Marks : 70

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Answer any five questions $\quad 5 \times 14=70$

1. a) A liquid from a reservoir is flowing down an inclined plane in thin film. The flow of the film is viscous and under the influence of gravity. Derive the equation for
i) velocity profile
ii) average velocity and
iii) volumetric flow rate using Navier-Stokes equation of motion.8
b) Derive the integral form of equation of continuity based on control volume approach, and hence show that for compressible fluid
$\left(\frac{D \rho}{D t}\right)=-\rho(\nabla . V)$
6
2. a) A fluid is moving between two infinitely broad parallel plates where both the plates are fixed. The flow is in

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$\mathrm{x}-\mathrm{y}$ plane and the fluid is moving in x -direction. Using Navier-Stokes equation for Newtonian fluid, obtain the expression for velocity distribution, maximum velocity and average velocity.
b) An incompressible velocity field is given by $\mathrm{u}=\mathrm{a}\left(\mathrm{x}^{2}-\mathrm{y}^{2}\right), \mathrm{w}=\mathrm{b}$ where $\mathrm{a}, \mathrm{b}$ are constant. What must the form of the velocity component be ? 5
3. Derive the equation for velocity profile for steady state viscous flow of fluid in a horizontal tube. The fluid is incompressible and viscosity is constant. The flow is driven in one direction by a constant pressure gradient.

For $z$ - component of equation of motion in cylindrical coordinates the Navier-Stokes equation is

$$
\begin{aligned}
& \rho\left(\frac{\partial V_{r}}{\partial t}+V_{r} \frac{\partial V_{z}}{\partial r}+\frac{V_{\theta}}{r} \frac{\partial V_{z}}{\partial \theta}+V_{z} \frac{\partial V_{z}}{\partial z}\right) \\
& =-\frac{\partial P}{\partial z}+\mu\left[\frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial V_{z}}{\partial r}\right)+\frac{1}{r} \frac{\partial^{2} V_{z}}{\partial \theta^{2}}+\frac{\partial^{2} V_{z}}{\partial z^{2}}\right]
\end{aligned}
$$

4. A Newtonian fluid is confined between two parallel vertical plates. The surface of the left plate is stationary and the other is moving vertically at a constant velocity $\mathrm{V}_{\mathrm{o}}$. Assuming that the flow is laminar obtain the equation for velocity profile.

The Navier-Stokes equation for the $y$-coordinate is

$$
\begin{aligned}
& \rho\left(\frac{\partial V_{y}}{\partial t}+V_{x} \frac{\partial V_{y}}{\partial x}+V_{y} \frac{\partial V_{y}}{\partial y}+V_{z} \frac{\partial V_{y}}{\partial z}\right) \\
& =-\frac{\partial P}{\partial y}+\mu\left(\frac{\partial^{2} V_{y}}{\partial x^{2}}+\frac{\partial^{2} V_{y}}{\partial y^{2}}+\frac{\partial^{2} V_{y}}{\partial z^{2}}\right)+\rho g_{y}
\end{aligned}
$$


5. For creeping flow between two concentric spheres (Figure given below) the $\theta$ component of the equation of motion for the system is as follows :

$$
0=-\frac{1}{r} \frac{\partial P}{\partial \theta}+\mu\left[\frac{1}{\sin \theta} \frac{1}{r^{2}} \frac{\mathrm{~d}}{\mathrm{~d} r}\left(r^{2} \frac{\mathrm{~d} u}{\mathrm{~d} r}\right)\right]
$$

Show that $\Delta P=B \ln \left(\frac{1-\cos \epsilon}{1+\cos \epsilon}\right)=-B E(\epsilon)$
where $\mathrm{B}=\sin \theta \frac{\mathrm{d} P}{\mathrm{~d} \theta}$ and $\Delta \mathrm{P}$ is the imposed overall drop in P .
Also, show that

$$
u=\frac{R \Delta P}{2 \mu E(\in)}\left[\left(1-\frac{r}{R}\right)+k\left(1-\frac{R}{r}\right)\right]
$$

and volumetric flow rate $Q=\frac{\Pi R^{3} \Delta P}{6 \mu E(\in)}(1-k)^{3}$


Creeping flow between two stationary concentric spheres

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6. Describe Boundary Layer Theory w.r.t. flow over a \&lat Plate and in a Circular Pipe.
7. a) Define Prandtl's Mixing Length
b) Show that for Bingham Flow in a Circular Tube
$v_{z} \geq \frac{\left(P_{o}-P_{L}\right) R^{2}}{4 \mu_{o} L}\left[1-\left(\frac{r}{R}\right)^{2}\right]-\frac{\tau_{o} R}{\mu_{o}}\left[1-\left(\frac{r}{R}\right)\right]$ for $r \geq r_{\mathrm{o}}$
and $v_{z} \leq \frac{\left(P_{o}-P_{L}\right) R^{2}}{4 \mu_{o} L}\left(1-\frac{r_{o}}{R}\right)^{2}$ for $r \leq r_{o}$

The radius and length of the tube are $R \& L$ respectively.
8. a) Describe the advantages and disadvantages of Fluidized Beds for Industrial Operations.
b) Discuss Segregation and Mixing in Fluidized Bed. 5

