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CS/M.TECH (CE)/SEM-1/CE-604/2012-13

2012

FINITE ELEMENT METHOD

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

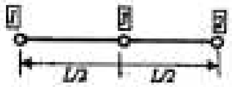
GROUP – A

Answer any *fifteen* of the following. $15 \times 2 = 30$

1. Explain with the stiffness matrix of a 2D solid element is symmetric ?
2. Explain why the global stiffness matrix generated by the finite element method is singular prior to the application of any boundary conditions ?
3. Select the right answer to the following statement :
   "The strain energy of an elastic structure calculated by the finite element method is".
   a) higher than the exact value
   b) equal to the exact value
   c) lower than the exact value

   and explain why ?
4. Explain why the shape functions needed for Bernoulli's beam formulation should be a polynomial of degree 3.

5. Draw the approximate shape functions for the truss element shown in the following figure.



Element 1

6. Explain why the element stresses across element boundaries are, in general, not in equilibrium ?

7. When is it possible to adopt the assumption of plane strain for the analysis of a three-dimensional structure ?

8. Why are linear quadrilateral elements not good candidates for flexural dominant two-dimensional elasticity problems ?

9. Explain how do you proceed to select the necessary required modes for a dynamic analysis of a structure in the frequency domain ?

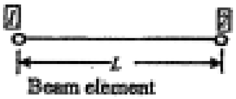
10. Explain how to select the time step for a dynamic analysis of a structure in the time domain.

11. Explain the source of errors in finite element modelling.

12. The solution of an elasticity problem obtained by the Finite Element Method does not guaranty equilibrium within the domain, explain why ?

13. Give the characteristics of the shape functions needed for the Bernoulli's beam formulation.

14. Draw the approximate shape functions for the Bernoulli's beam element show in the following figure.

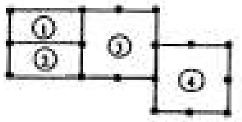


15. Explain the relationship between the number of integration pints and the order of a 2D elasticity finite element.

16. The interpolation functions for the four-nodes quadrilateral element verify the following condition :

$\sum_{i=1}^{4} N_i(\xi, \eta) = 1$ for any couple (ξ, η), what is the significance of this observation ?

17. Identify the defects associated with connecting four-node and eight-node elements in the pattern shown



18. Explain why linear quadrilateral elements are not good candidates for flexural domain two-dimensional elasticity problem ?

19. Finite element analysis gives stresses in general coordinate directions in terms of σ_x, σ_y, etc. Discuss how you can interpret these results for ductile (e.g. mild steel) and brittle (e.g. concrete) materials.

20. What happens if the Poisson's ratio, ν, becomes close to 0·5 in the plane strain case ? What is the implication in modelling such materials using the finite element method ?

GROUP – B

Answer any *two* of the following. $2 \times 10 = 20$

21. For the plane strain element shown in Figure 1, determine the element stresses σ_x, σ_y and τ_{xy} corresponding to the following nodal displacements :

$$u_1 = 5 \cdot 0 \text{ mm } \upsilon_1 = 2 \cdot 0 \text{ mm}$$

$$u_1 = 0 \cdot 0 \text{ mm } \upsilon_2 = 0 \cdot 0 \text{ mm}$$

$$u_3 = 5 \cdot 0 \text{ mm } \upsilon_3 = 0 \cdot 0 \text{ mm}$$

use E = 70GP an and υ = 0·3

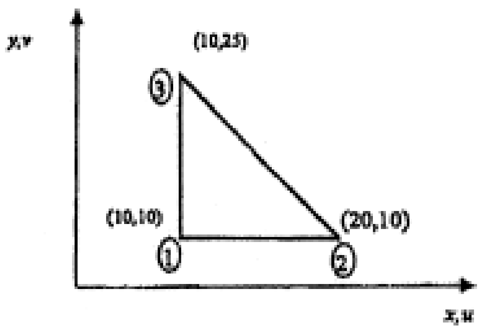
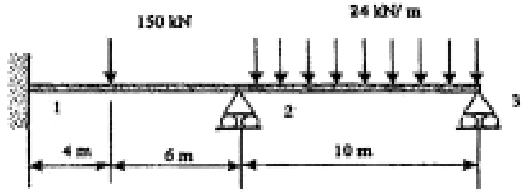


Fig. 1

22. Using the finite element method, calculate the reactions and draw the shear and moment diagrams for the structure shown below (Figure 3). In addition to the illustrated loading, support 2 is subjected to a settlement of 12 mm downward.

All members have the same rigidity EI-70000kN.m^2. The stiffness matrix of a beam element is given by

$$[k] = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$



23. a) Show that the international of the displacement field inside the rectangular element, illustrated in Fig 2(a), is defined by $u\,(x,y) = \sum_{in1}^{4} N_1\,(x,\,y)\,u_i,$ where N_i are given by

$$N_1\,(x,y) = \frac{(a-x)(b-y)}{4ab} \qquad N_3\,(x,y) = \frac{(a+x)(b+y)}{4ab}$$

$$N_2\,(x,y) = \frac{(a+x)(b-y)}{4ab} \qquad N_4\,(x,y) = \frac{(a-x)(b+y)}{4ab}$$

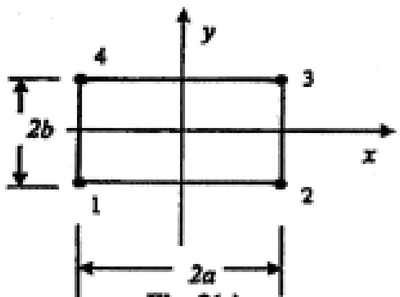


Fig. 2(a)

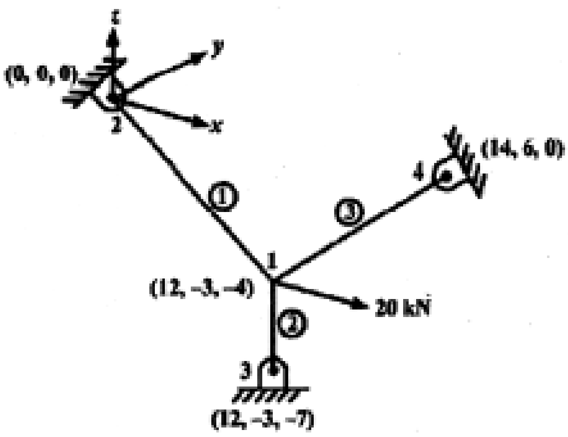
b) The above illustrated four nodes rectangular element is made from an elastic material with an elasticity modulus E and a Poisson's ratio υ and has a thickness t. Show that in the case of plane stress, the stiffness term K_{33} is given by

$$k_{33} = \left(\frac{b}{3a} + \frac{1-\upsilon}{6} \frac{a}{b} \right) \frac{Et}{1-\upsilon^2}$$

GROUP – C

Answer any *one* of the following. $1 \times 20 = 20$

24. Analyze the space truss shown in the figure below. The truss is composed of four nodes, whose coordinates (in meters) are shown in the figure, and three elements, whose cross-sectional area are all $10 \times 10^{-4}\ \text{m}^2$. The modulus of elasticity E = 210 GPa for all the elements. A load of 209 kN in applied at node 1 in the global x-direction. Nodes 2 to 4 are pin supported and thus constrained form movement in the *x*, *y*, and z directions.



25. A beam is clamped at the left end and on a spring at the right end as shown in the figure below. A force F = 3,000 N acts downward at the right end as shown. The spring stiffness k = 3,000 N/m. The beam properties are : L = 1 m, EI = 1,000 Nm^2. Determine the deflection curve $v(x)$ and bending moment curve $M(x)$.