Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/M.TECH (BT)/SEM-1/MBT-104/2010-11
2010-11
NUMERICAL ANALYSIS & BIOSTATISTICS

Time Allotted : 3 Hours   *Full Marks* : 70

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

GROUP – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following :

$$10 \times 1 = 10$$

   i) The probability that the 4 children of a family have different birthdays is

   a) 0·9836   b) 0·4735

   c) 0·9   d) 0·75

   ii) If $2x = 4y + 7$ be a regression line of x on y, then b_{xy} is

   a) $\dfrac{1}{2}$ b) 2

   c) 4   d) 1.

   iii) If two variables are uncorrelated, then r_{xy} is

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a)   0   b)   1

c)   2   d)   3.

iv)   The distribution for which mean and variance are equal is

a)   Exponential   b)   Binomial

c)   Normal   d)   Poisson.

v)   Which of the following is true for random variable X, where a, b are arbitrary constants ?

a) $E\,(\,aX + b\,) = aE\,(\,X\,)$

b) $Var\,(\,aX + b\,) = b^2\,Var\,(\,X\,)$

c) $E\,(\,aX + b\,) = b$

d) $Var\,(\,aX + b\,) = a^2\,Var\,(\,X\,)$.

vi)   Round-off the number 9·478556, correct up to 4 decimal places is

a)   9·4785   b)   9·4795

c)   9·4786   d)   9·4756.

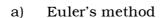vii)   For Trapezoidal Rule, the number of quadrature points is
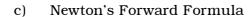
a)   one   b)   two

c)   three   d)   four.

viii)   ………………… is are used to solve the Numerical solution of O.D.E. of first order.

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a) Euler's method

b) Simpson's $\frac{1}{3}$ Rule

c) Newton's Forward Formula

d) Hermite Polynomials.

ix) For a bivariate data (x, y), the correlation coefficient r_{xy} lies between

a) $-1 \leq r_{xy} \leq 1$ b) $-\infty \leq r_{xy} \leq \infty$

c) $0 \leq r_{xy} \leq 1$ d) $-1 \leq r_{xy} \leq 0$.

x) A function $f(x)$ is said to be probability density function if

a) $\int_{-\infty}^{x} f(x)\, \mathrm{d}x = 1$ b) $\int_{0}^{\infty} f(x)\, \mathrm{d}x = 1$

c) $\int_{-\infty}^{\infty} f(x)\, \mathrm{d}x = 1$ d) $\int_{x}^{\infty} f(x)\, \mathrm{d}x = 1$.

xi) A statistics is a function of sample observations.

a) True   b) False.

xii) Which of the following is type II error ?

a) The error of accepting H_0 when H_0 is true

b) The error of rejecting H_0 when H_0 is false

c) The error of accepting H_0 when H_0 is false

d) The error of rejecting H_0 when H_0 is true.

xiii) In testing of hypothesis, type I and type II errors are complementary to each other.

a) True   b)   False.

xiv) Null hypothesis is in terms of

a)   Sample   b)   Constant

c)   Parameter   d)   Statistic.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2.   Find the mean and standard deviation of a binomial distribution.

3.   Evaluate $\int_{0}^{5} \dfrac{dx}{1 + x}$, by Trapezoidal Rule, taking $h = 1$.

4.   Given that $\dfrac{dy}{dx} = x + y$, with the initial condition $y (0) = 1$. Find $y (0.5)$, correct up to two decimal places, taking step length $h = 0.1$.

5.   For any bivariate data (x, y), prove that $-1 \leq r_{xy} \leq 1$, where r_{xy} is the correlation coefficient of x and y.
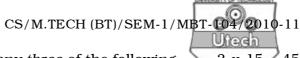
6.   Show that $f (x)$ given by

$f (x) = x ;\ 0 < x < 1$

$= k - x ;\ 1 < x < 2$

$= 0 ;\ \text{elsewhere}.$

is a probability density function for a suitable value of k. Calculate $P \left(\dfrac{1}{2} \leq X \leq \dfrac{3}{2} \right)$.

GROUP – C

(Long Answer Type Questions)

7. a) The probability density function of a random variable

X is

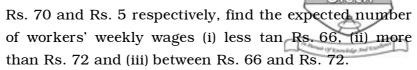$$f(x) = k(x-1)(x-2); \quad 1 \le x \le 2$$

$$= 0, \quad \text{elsewhere.}$$

Determine —

i) the value of the constant k

ii) the distribution function $F(x)$

iii) $P\left(\dfrac{5}{4} \le X \le \dfrac{3}{2}\right)$.

b) The relationship between travel expenses (y) and the duration of travel (x) is found to be linear. A summary of data for 102 pairs is given below :

$$\sum x = 510, \ \sum y = 7140, \ \sum x^2 = 4150,$$

$$\sum xy = 54,900 \text{ and } \sum y^2 = 7,40,200.$$

i) Find the two regression coefficients.

ii) Find the two regression equations.

iii) A given trip has to take seven days. How much money should a salesman be allowed so that he will not run short of money ? 7 + 8

8. a) If the weekly wage of 10,000 workers in a factory follows normal distrituion with mean and standard deviation

Rs. 70 and Rs. 5 respectively, find the expected number of workers' weekly wages (i) less tan Rs. 66, (ii) more than Rs. 72 and (iii) between Rs. 66 and Rs. 72.

$$\left[\begin{array}{l} \text{Given that } \dfrac{1}{\sqrt{2\pi}} \int\limits_{0}^{0\cdot4} e^{-\frac{t^2}{2}} \, dt = 0\cdot1554 \text{ and} \\[2em] \dfrac{1}{\sqrt{2\pi}} \int\limits_{0}^{0\cdot8} e^{-\frac{t^2}{2}} \, dt = 0\cdot2881 \end{array}\right]$$

b) Solve by Euler's modified method the following differential equation for $x = 0\cdot02$, by taking step length $h = 0\cdot01$, $\dfrac{dy}{dx} = x^2 + y$, $y = 1$ when $x = 0$.   7 + 8

9. a) Find $f'(1)$, $f''(1)$, $f'(6)$ and $f''(6)$ for the function $y = f(x)$ given in the table :

x :	1	2	3	4	5	6
y :	2·7183	3·3210	4·0552	4·9530	6·0496	7·3891

b) Compute $y(0\cdot2)$, by Runge-Kutta method, correct up to two decimal places, from the equation $\dfrac{dy}{dx} = xy$, $y(0) = 2$, taking $h = 0\cdot2$.   10 + 5

10. a) Calculate the Quartile deviation from the following :

Class-interval :	10-15	15-20	20-25	25-30
Frequency :	4	12	16	22

30-40	40-50	50-60	60-70
10	8	6	4

b) Compute the standard deviation of household size from the following frequency distribution of 500 households :

Household size :	1	2	3	4	5	6	7	8	9
No. of Households :	92	49	52	82	102	60	35	24	4

c) You are given below the wages paid to some workers in a small factory. Form a frequency distribution with class-interval 10 paise :

Wages in Rs. :

1·10 1·13 1·44 1·27 1·17 1·98 1·36 1·30 1·44

1·27 1·24 1·73 1·51 1·12 1·42 1·03 1·58 1·46

1·40 1·21 1·62 1·31 1·55 1·33 1·04 1·48 1·20

1·60 1·70 1·09 1·49 1·86 1·95 1·51 1·82 1·42

1·29 1·54 1·38 1·87 1·41 1·77 1·15 1·57 1·07

1·65 1·36 1·67 1·41 1·55 1·22 1·69 1·67 1·34

1·45 1·39 1·25 1·26 1·75 1·57 1·53 1·37 1·59

1·19 1·52 1·56 1·32 1·81 1·40 1·47 1·38 1·62

1·76 1·28 1·92 1·46 1·46 1·35 1·16 1·42 1·78

1·68 1·47 1·37 1·35 1·47 1·43 1·66 1·56 1·48

5 + 5 + 5

11. a) Use the sign test to see if there is a difference between the number of days required to collect an account

receivable before and after a new collection policy. Use the 0·05 significance level.

Before :   33 36 41 32 39 47 34 29 32 34 40 42 33 36 27

After :   35 29 38 34 37 47 36 32 30 34 41 38 37 35 28

b)   Calculate the value of $\int_0^1 \dfrac{x\,\mathrm{d}x}{1+x}$, correct up to two decimal places, taking six intervals by (i) Simpson's One-third Rule, (ii) Trapezoidal Rule.   8 + 7

12. a)   Define the Type I error and Type II error.

b)   In order to test whether a coin is perfec, the coin is tossed 5 times. The null hypothesis of perfectness is rejected if more than 4 heads are obtained. What is the probability of Type I error ? Find the probability of Type II error when the corresponding probability of head is 0·2.

c)   Survey of 320 families with 5 children each revealed the following distribution :

No. of Boys :	5	4	3	2	1	0
No. of Girls :	0	1	2	3	4	5
No. of Family :	14	56	110	88	40	12

Is the result consistent with the hypothesis that male and female births are equal probable. The 5% value of χ^2 with 5 degree of freedom is 11·07.   3 + 5 + 7