Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/M.Tech (AEIE)/SEM-1/EIEM-102/2012-13

2012
SIGNAL AND SYSTEMS

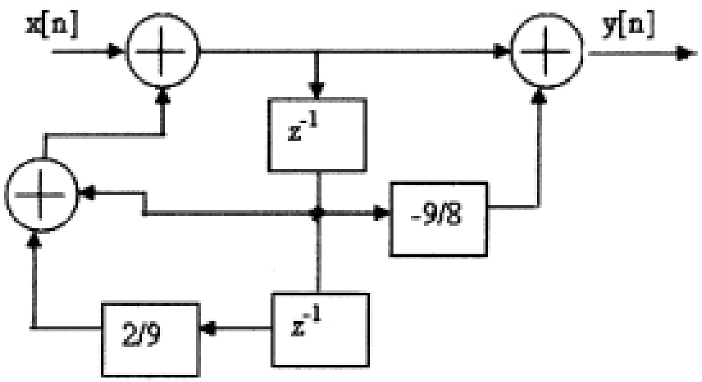
Time Allotted : 3 Hours | *Full Marks* : 70

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Answer any *five* of the following $5 \times 14 = 70$

1. a) Find the convolution of two finite duration sequences $h(n) = a^n u(n)$ and $x(n) = b^n u(n)$ for all n when $a \neq b$. 4

b) The block diagram representation of an LTI system is shown below. Determine the difference equation relating $y(n)$ and $x(n)$. 6



c) An LTI system is characterized by the system function $H(z) = \dfrac{3-4z-1}{1-3\cdot 5z^{-1}+1\cdot 5z^{-1}}$.

Specify the ROC of $H(z)$ and determine $h(n)$ for the following conditions :

i) System is stable

ii) System is causal 4

2. a) A causal LTI system with input $x(n)$ and output $y(n)$ is described by the following difference equation.

$y(n) = y(n-1) + y(n-2) + x(n-1)$

i) Plot the poles and zeros of the system

ii) Find the unit sample response of the system.

iii) Comment on the stability of the system with justification. 2 + 4 + 2

b) Find inverse z-transform of $X(z) = \dfrac{z(z^2-4z+5)}{(z-1)(z-2)(z-3)}$

for ROC

(i) $2 < |z| < 3$

(ii) $|z| > 3$

(iii) $|z| < 1$ 6

3. a) What is zero padding ? Mention the application of zero padding. 2

b) Find out the output of a system with inpulse response $h(n) = \{1, -3, 4, 0, 2\}$ subjected to the input sequence $x(n) = \{3, -1, 4\}$ using circular convolution method. 6

c) Find the output of a system with impulse response $h(n) = \{1, 2\}$ for an input sequence $x(n) = \{1, 2, -1, 2, 3, -2, -3, -1, 1, 1, 2, 1\}$ using overlap-add method. 6

4. a) Find the DFT of a sequence $x(n) = \{1, 2, 3, 4, 4, 3, 2, 1\}$ using radix 2 DIT-FFT algorithm. 6

b) A system is characterized by the impulse response $h(n) = \left(\frac{1}{2}\right)^n u(n)$. Determine the output sequence subjected to the input sequence

$x(n) = 10 - 5 \sin\left(\frac{\pi}{2} n\right) + 20 \cos(\pi n)$ when $-\infty < n < \infty$

8

5. a) What is warping effect in IIR filter design using Bilinear Transform ? How can it be eliminated ? 3

b) Determine the order and poles of a type I lowpass Chebyshev filter that has a 1 dB ripple in the pass band and the pass band frequency $\Omega_p = 1000\pi$, a stop band frequency 2000π and an attenuation of 40 dB or more. 4

c) Design a high pass filter monotonic in pass band with cut-off frequency 1000 Hz and down 10 dB at 350 Hz using bilinear transform. Sampling frequency is 5 kHz.

7

6. a) Realize the system given be the difference equation

$y(n) = -0{\cdot}1\, y(n-1) + 0{\cdot}93 y(n-2) + 0{\cdot}6 x(n) - 0{\cdot}543 x(n-2)$

in parallel form. 5

b) A two-pole low pass filter has system function $H(z) = \frac{b_0}{(1 - pz^{-1})^2}$. Determine the value of b_0 and p such that the frequency response satisfies the condition $H(0) = 1$ and $\left|H\left(\frac{\pi}{4}\right)\right|^2 = \frac{1}{2}$ 4

c) Design a FIR filter of length 3 for an input signal $x(n) = \cos(0\cdot 1n) + \cos(0\cdot 4n)$ to retain the higher frequency. 5

7. a) Design an ideal high pass FIR filter with frequency response

$$H_d(e^{j\omega}) = \begin{cases} 1 & \text{for } \frac{\pi}{4} \le \omega \le \pi \\ 0 & \text{for } \omega \le \frac{\pi}{4} \end{cases}$$

Find the value of $h(n)$ for $N = 11$. 8

b) Derive state space representation of the causal LTI system given below : 6

$$y(n) = -\sum_{k=1}^{4} a_k\, y(n-k) + \sum_{k=0}^{4} b_k\, x(n-k)$$

8. a) The state model of a system is given by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -3 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} [\,u\,]$$

Find

i) the characteristic equation and eigenvalues of the system

ii) the transfer function $\dfrac{Y(s)}{U(s)}$

iii) the state transition matrix

iv) the state equation for a unit step input under zero initial conditions. 9

b) Realize the first order transfer function $H(z) = \dfrac{1}{1 - az^{-1}}$ and draw its quantization noise model. Find the steady state noise power due to product round-off. 5