Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/M.Tech (AEIE)/SEM-1/EIEM-102/2011-12

2011

SIGNALS AND SYSTEMS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer any *five* questions. 5 × 14 = 70

1. a) Determine the response of the system with impulse response $h(n)=\left(\frac{1}{2}\right)^n u(n)$ subjected to the input signal $x(n)=10-5\sin\frac{\pi}{2}n+20\cos\pi n$. 5

   b) Prove that discrete time sinusoids whose frequencies are separated by an integer multiple of 2π are identical. 2

   c) Determine the power and energy of unit step sequence. 4

   d) Is the system described by $y(n)=\frac{x^2(n)}{x(n-1)}$ linear ? 3

2. a) An LTI system has $H(z)=\frac{1-2z^{-1}}{1-1\cdot 25\,z^{-1}}$. Is the system FIR or IIR ? 3

b) Find out the inverse Z transform of $X(z) = \dfrac{z+2}{2Z^2 - 7z + 3}$ using PFE method. If the ROCs are

i) $|z| > 3$

ii) $|z| < \dfrac{1}{2}$

iii) $\dfrac{1}{2} < |z| < 3$. 5

c) Two causal LTI systems having impulse responses $h_1(n)$ and $h_2(n)$, respectively, are cascaded as follows :

$$x(n) \rightarrow \boxed{h_{1(n)}} \rightarrow \boxed{h_{2(n)}} \rightarrow y(n)$$

If $h_1(n) = \left(\dfrac{1}{2}\right)^n u(n)$ and $h_2(n) = \left(\dfrac{1}{4}\right)^n u(n)$, determine the impulse response of the total system. 6

3. a) Find the 8-point DFT using decimation in time FFT algorithm for a sequence $x(n) = \{1, 3, 5, 7, 2, 4, 6, 8\}$ 6

b) Perform the linear convolution of the sequences $x_1(n) = (1, 2, 3\ -4)$ and $x_2(n) = (2, -3, -1, 6)$ using circular convolution method. 5

c) What is the relation between DFT and Z-transform ? 3

4. a) Determine the solution of the difference equation :

$$y(n) = \frac{5}{6} y(n-1) - \frac{1}{6} y(n-2) + x(n) \text{ for } x(n) = 2^n u(n)$$ if the initial condition is given as $y(-1) = y(-2) = 0$. 6

b) An LTI system is characterized by the system function $H(z) = \dfrac{3 - 4z^{-1}}{1 - 3\cdot5\,z^{-1} + 1\cdot5\,z^{-2}}$. Specify the ROC of $H(z)$ and determine $h(n)$ for the following conditions :

i) System is stable

ii) System is anticausal. 4

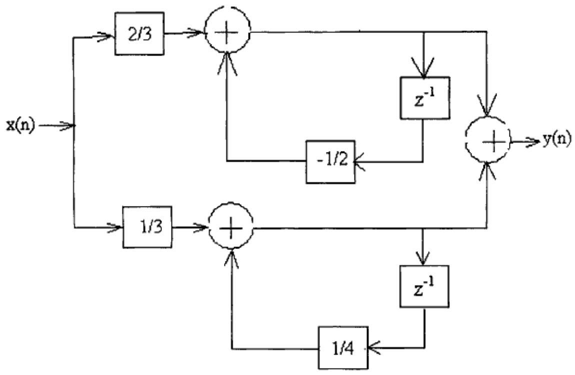
c) If $X(z) = Z\{x(n)\}$, then prove that $Z\{nx(n)\} = -\dfrac{d}{dz}X(z)$. 4

5\. a) An analog filter transfer function is given by $H(s) = \dfrac{1}{(s+3)(s+5)}$. Find the transfer function $H(z)$ of IIR digital filter using impulse invariable transformation. Assume $T = 0\cdot1$ sec and draw the realization. 6

b) Realize the system with difference equation $y(n) = \dfrac{3}{4}y(n-1) - \dfrac{1}{8}y(n-2) + x(n) + \dfrac{1}{3}x(n-1)$ in cascade form. 5

c) What is the advantage of the digital filters over analog filters ? 3

6\. a) The block diagram representation of an LTI system is shown in the figure below :



Determine the difference equation of the system relation $y(n)$ and $x(n)$. Comment on the stability of the system with justification. 5

b) Determine an sketch the magnitude and phase response of the system $y(n) = \frac{1}{2}\left[x(n) + x(n-2)\right]$. 5

c) By DFT and IDFT method, determine the response of FIR filter with impulse response $h(n) = \{1, 2, 3\}$ to the input sequence $x(n) = \{1, 2, 2, 1\}$. 4

7. a) Find the steady state variance of the noise in the output due to quantization of input for the first order filter given by $y(n) = \{1, 2, 2, 1\}$. 4

b) What is product roundoff error in digital signal processing ? 2

c) Consider the transfer function $H(z) = H_1(z) H_2(z)$ where $H_1(z) = \dfrac{1}{1 - a_1 z^{-1}}$ and $H_2(z) = \dfrac{1}{1 - a_2 z^{-1}}$. Find the output roundoff noise power for $a_1 = 0 \cdot 5$ and $a_2 = 0 \cdot 6$. 8

8. a) Find the correlation coefficient of X and Y if

$$f(x, y) = x + y \text{ for } 0 < x \le 1;\ 0 < y \le 1$$
$$= 0 \text{ otherwise.}$$
7

b) Determine the system function $H(z)$ and the impulse response $h(n)$ of the system described by the following state space equation :

$$v(n+1) = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} v(n) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} x(n)$$

and $y(n) = \begin{bmatrix} 1 & 1 \end{bmatrix} v(n) + x(n)$. 7