*Name* : ..................................................................

*Roll No.* : ..............................................................

*Invigilator's Signature* : ..............................................

**CS/M.TECH(AEIE)/SEM-1/EIEM-100(AM)/2011-12**

**2011**

# ADVANCED ENGINEERING MATHEMATICS

*Time Allotted* : 3 Hours *Full Marks* : 70

*The figures in the margin indicate full marks.*

Answer any ***five*** questions. 5 × 14 = 70

1. a) Find the analytic function $f(z) = u + iv$ whose imaginary part is $v = e^x \sin y$. 7

b) Evaluate $\int_C \bar{z}\, dz$ from $z = 0$ to $z = 4 + 2i$ along the curve $C$ given by

i) the straight line joining $z = 0$ to $z = 4 + 2i$

ii) along the straight line from $z = 0$ to $z = 2i$

iii) along the straight line from $z = 2i$ to $z = 4 + 2i$

2 + 2 + 3

2. a) Expand $f(z) = \dfrac{1}{1+2z}$ in a Taylor series about the origin. 7

b) Evaluate $\displaystyle\int_c \frac{z^2+6z-1}{z-4}\,dz$ if $c$ is

i) $c : |\ z\ | = 5$

ii $c : |\ z\ | = 1/2$ 3 + 4

3. a) Prove that $\lim_{(x,y)\to(0,0)} \dfrac{xy}{x^2+y^2}$ does not exist. 4

b) Verify that the double limit $\lim_{(x,y)\to(0,0)} \dfrac{x+y}{x-y}$ does not exist. Prove that repeated limits exist. 5

c) Let $F(x,y) = x^2+y^2-1$ and a point ( 0, 1). Verify implicit function theorem. 5

4. a) Examine the curve $y^2(1+x) = x^2(1-x)$ for singular points at the origin. 4

b) Examine the maxima and minima of the function

$f(x,y) = 2x^2 - xy + 2y^2 - 20x$ 4

c) If $u = a^3x^2 + b^3y^2 + c^3z^2$, where $\dfrac{1}{x}+\dfrac{1}{y}+\dfrac{1}{z} = 1$, prove that a stationary value is given by $ax = by = cz$ and this gives a maximum or a minimum, if $abc\,(a+b+c)$ is positive. Solve the problem by Lagrange's multiplier method. 6

5. a) Use Lagrange's interpolation formula to fit a polynomial to the following data. Hence find the value of $y$ for $x$ = 5·4, correct to 5 significant figures. 7

| | | | | | |
|---|---|---|---|---|---|
| $x$ : | 5·0 | 5·1 | 5·3 | 5·5 | 5·6 |
| $y$ : | 0·28366 | 0·37798 | 0·55437 | 0·70867 | 0·77557 |

b) Construct the difference table and find the value of $y$ ( 3, 4) using Newton's divided difference formula from the following table : 7

| | | | | | |
|---|---|---|---|---|---|
| $x$ : | 2·5 | 2·8 | 3·0 | 3·1 | 3·6 |
| $y$ : | 12·1825 | 16·4446 | 20·0855 | 22·1980 | 36·5982 |

6. a) Find the root of the equation $xe^x - 3 = 0$ that lies between 1 and 2, correct to 4 significant figures using the method of false position. 7

b) Compute $y$ (0·4), from the differential equation $\frac{dy}{dx} = x - y, \ \ y(0) = 1$, taking $h$ = 0·1, by Runge-Kutta method, correct to 5 decimal places. 7

7. a) Let $V$ be a real vector space with { $a, b, c$ } as a basis. Prove that the set { $a + b + c, b + c, c$ } is also a basis of $V$. 6

b) Find a basis and the dimension of the subspace $W$ of $R^3$ where $W = \{ ( x, y, z ) \in R^3 : x + 2y + z = 0, \ 2x + y + 3z = 0 \}$ 4

c) Check whether the vectors (1, 1, 1), ( 2, –1, 0 ) and ( 5, 3, 9 ) are linearly independent in $R^3$. 4

8. a) Prove that if $\lambda$ is an eigenvalue of a non-singular matrix $A$, then $\frac{1}{\lambda}$ is an eigenvalue of $A^{-1}$. 4

b) Determine whether $S = \{ (x, y, z) : x + 2y + 5z = k,\ k \neq 0 \}$ is a subspace of $R^3$. Justify. 6

c) Prove that if $S_1$ and $S_2$ are subspaces of a vector space $V$, then $S_1 \cap S_2$ is also a subspace of $V$. 4

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