



Name :

Roll No. :

Invigilator's Signature :

CS/M.TECH(AEIE)/SEM-1/EIEM-100(AM)/2011-12

2011

ADVANCED ENGINEERING MATHEMATICS

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

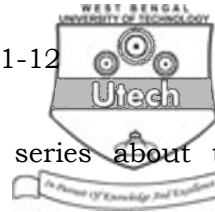
Answer any **five** questions. $5 \times 14 = 70$

1. a) Find the analytic function $f(z) = u + iv$ whose imaginary part is $v = e^x \sin y$. 7

- b) Evaluate $\int_C \bar{z} dz$ from $z = 0$ to $z = 4 + 2i$ along the curve C given by

- i) the straight line joining $z = 0$ to $z = 4 + 2i$
- ii) along the straight line from $z = 0$ to $z = 2i$
- iii) along the straight line from $z = 2i$ to $z = 4 + 2i$

$2 + 2 + 3$



2. a) Expand $f(z) = \frac{1}{1+2z}$ in a Taylor series about the origin. 7

- b) Evaluate $\int_c \frac{z^2 + 6z - 1}{z - 4} dz$ if c is

i) $c : |z| = 5$

ii) $c : |z| = 1/2$ 3 + 4

3. a) Prove that $\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^2 + y^2}$ does not exist. 4

- b) Verify that the double limit $\lim_{(x,y) \rightarrow (0,0)} \frac{x+y}{x-y}$ does not exist. Prove that repeated limits exist. 5

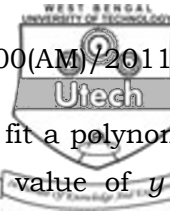
- c) Let $F(x, y) = x^2 + y^2 - 1$ and a point $(0, 1)$. Verify implicit function theorem. 5

4. a) Examine the curve $y^2(1+x) = x^2(1-x)$ for singular points at the origin. 4

- b) Examine the maxima and minima of the function

$$f(x, y) = 2x^2 - xy + 2y^2 - 20x$$
 4

- c) If $u = a^3x^2 + b^3y^2 + c^3z^2$, where $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$, prove that a stationary value is given by $ax = by = cz$ and this gives a maximum or a minimum, if $abc(a+b+c)$ is positive. Solve the problem by Lagrange's multiplier method. 6



5. a) Use Lagrange's interpolation formula to fit a polynomial to the following data. Hence find the value of y for $x = 5.4$, correct to 5 significant figures. 7

$x:$	5.0	5.1	5.3	5.5	5.6
$y:$	0.28366	0.37798	0.55437	0.70867	0.77557

- b) Construct the difference table and find the value of $y(3.4)$ using Newton's divided difference formula from the following table : 7

$x:$	2.5	2.8	3.0	3.1	3.6
$y:$	12.1825	16.4446	20.0855	22.1980	36.5982

6. a) Find the root of the equation $xe^x - 3 = 0$ that lies between 1 and 2, correct to 4 significant figures using the method of false position. 7

- b) Compute $y(0.4)$, from the differential equation $\frac{dy}{dx} = x - y$, $y(0) = 1$, taking $h = 0.1$, by Runge-Kutta method, correct to 5 decimal places. 7

7. a) Let V be a real vector space with $\{a, b, c\}$ as a basis. Prove that the set $\{a + b + c, b + c, c\}$ is also a basis of V . 6

- b) Find a basis and the dimension of the subspace W of R^3 where $W = \{(x, y, z) \in R^3 : x + 2y + z = 0, 2x + y + 3z = 0\}$ 4

- c) Check whether the vectors $(1, 1, 1)$, $(2, -1, 0)$ and $(5, 3, 9)$ are linearly independent in R^3 . 4



8. a) Prove that if λ is an eigenvalue of a non-singular matrix A , then $\frac{1}{\lambda}$ is an eigenvalue of A^{-1} . 4
- b) Determine whether $S = \{ (x, y, z) : x + 2y + 5z = k, k \neq 0 \}$ is a subspace of R^3 . Justify. 6
- c) Prove that if S_1 and S_2 are subspaces of a vector space V , then $S_1 \cap S_2$ is also a subspace of V . 4

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