

CS/MCA/SEM-4/MM-401/2012
2012
OPERATIONS RESEARCH \& OPTIMIZATION TECHNIQUES
Time Allotted: 3 Hours
Full Marks : 70

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Graph sheet(s) will be supplied by the Institution on demand.

## GROUP - A

( Multiple Choice Type Questions )

1. Choose the correct alternatives for any ten of the following :

$$
10 \times 1=10
$$

i) The solution to a transportation problem with $m$ rows and $n$ columns is feasible if number of positive allocations are
a) $m+n$
b) $m \times n$
c) $m+n-1$
d) $m+n+1$.
ii) The possible number of basic solutions in a system of $m$ equations with $n$ unknowns is
a) $\frac{n!}{m!(n-m)!}$
b) $m n$
c) $(n+m)$ !
d) none of these.
iii) If $x$ is a convex combination of $y$ and $z$ and $x=a y+b z$, then
a) $a+b=1$
b) $a+b=0$
c) $a=b$
d) none of these.
iv) The role of artificial variable in the simplex method is
a) to aid in finding an initial solution-
b) to find optimal dual prices in the final simplex table
c) to start phases of simplex method
d) all of these.
v) Every extreme point of the convex set of all feasible solutions of the system $A x=b, x \geq 0$ corresponds to
a) a basic solution
b) a feasible solution
c) both (a) and (b)
d) none of these.
vi) If a primal linear programming problem has unbounded solution, then its corresponding dual will have
a) unbounded solution
b) unique solution
c) no feasible solution
d) none of these.
vii) If in assignment problem, ' $k$ ' be the maximum number of zeroes which can be assigned, then the minimum number of lines which will cover all the zeroes is
a) $k$
b) $k+1$
c) $k-1$
d) $2 k$.
viii) For a two person game with $A$ and $B$, the minimizing and the maximizing players, the optimum strategies are
a) minimax for $A$ and maximin for $B$
b) maximin for $A$ and minimax for $B$
c) minimin for $A$ and maximin for $B$
d) maximin for $A$ and minimin for $B$.
ix) The formula for minimum inventory cost under purchasing model without shortage is
a) $\sqrt{2 R C_{1} C_{3}}$
b) $\sqrt{2 C_{3} R / C_{1}}$
c) $\quad \sqrt{G} /\left(2 R C_{3}\right)$
d) none of these.
x) In a PERT network, the starting vertex is a
a) burst node
b) merge node
c) $\operatorname{root}$
d) none of these.
xi) CPM is

xii) If $\lambda$ is the arrival rate, $\mu$ is the service rate, then the average waiting time of a customer (in the queue) is given by
a) $\frac{\lambda}{[\lambda(\mu-\lambda)]}$
b) $\frac{\lambda}{[\lambda(\mu+\lambda)]}$
c) $\frac{\lambda}{(\mu-\lambda)}$
d) $\frac{1}{\lambda(\mu-\lambda)}$.
xiii) The objective of network analysis is to
a) minimize total project duration
b) minimize total project cost
c) minimize production delays, interruption and conflicts
d) all of these.

## GROUP - B

( Short Answer Type Questions)
Answer any three of the following. $3 \times 5=15$
2. Solve the following LPP by graphical method :

Maximize $Z=3 x_{1}-x_{2}$
subject to $2 x_{1}+x_{2} \geq 2$,

$$
x_{1}+3 x_{2} \leq 2,
$$

$$
x_{2} \leq 4,
$$

$$
x_{1}, x_{2} \geq 0 .
$$

3. Use the simplex method to solve the following LP problem :

Minimize $Z=600 x_{1}+500 x_{2}$
subject to the constraints

$$
\begin{aligned}
2 x_{1}+x_{2} & \geq 80 \\
x_{2}+2 x_{3} & \geq 60 \\
\text { and } x_{1}, x_{2} & \geq 0 .
\end{aligned}
$$

4. For the game with payoff matrix :

Player $B$


| Player $A$ | $B_{1}$ | $B_{2}$ | $B_{3}$ |
| :---: | :---: | :---: | :---: |
| $A_{1}$ | -1 | 2 | -2 |
| $A_{2}$ | 6 | 4 | -6 |

Determine the optimal strategies for players $A$ and $B$. Also determine the values of game.
5. A company has three plants at locations A, B and C, which supply to warehouses located at D, E, F, G and H. Monthly plant capacities are 800,500 and 900 units respectively. Monthly warehouse requirements are 400, 400, 500, 400 and 800 units respectively. Unit transportation costs (in rupees) are given in the adjacent table .

| F | $\downarrow$ | $\downarrow$ | TO |  |  |  |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- |
|  | $\downarrow$ | $D$ | $E$ | $F$ | $G$ | $H$ |
| O | $A$ | 5 | 8 | 6 | 6 | 3 |
| M | $B$ | 4 | 7 | 7 | 6 | 6 |
|  | C | 8 | 4 | 6 | 6 | 3 |

Determine an initial basic feasible solution of the problem using VAM. Check whether the solution obtained is degenerate or not.
6. The production department for a company requires 3600 kg of a raw material for manufacturing a particular item per year. It has been estimated that the cost of placing an order is Rs. 36 and the cost of carrying inventory is 25 per cent of the investment in the inventories. The price is Rs. 10 per kg. Find the minimum total annual inventory cost.

7. a) Use dynamic programming to find the value of $\operatorname{Max} Z=y_{1} y_{2} y_{3}$
subject to constraints :

$$
\begin{align*}
& y_{1}+y_{2}+y_{3}=5 \\
& y_{1}, y_{2}, y_{3} \geq 0 \tag{8}
\end{align*}
$$

b) Find the dual of the following primal problem :

Maximize $Z=2 x_{1}+3 x_{2}+4 x_{3}$
subject to $x_{1}-5 x_{2}+3 x_{3}=7$

$$
\begin{align*}
& 2 x_{1}-5 x_{2} \leq 3 \\
& 3 x_{2}-x_{3} \geq 5 \\
& x_{1}, x_{2} \geq 0 \tag{7}
\end{align*}
$$

and $x_{3}$ is unrestricted in sign.
8. a) Find the optimal assignments to find the minimum cost for the assignment problem with the following cost matrix :

|  | A | B | C | D |
| :--- | :---: | :---: | :---: | :---: |
| 1 | 18 | 26 | 17 | 11 |
| 2 | 13 | 28 | 14 | 26 |
| 3 | 38 | 19 | 18 | 15 |
| 4 | 19 | 26 | 24 | 10 |
|  |  |  |  |  |

b) Solve the following linear integer programmingaproblem using 'cutting plane method' :


Maximize $Z=14 x_{1}+16 x_{2}$
subject to constraints

$$
\begin{aligned}
& 4 x_{1}+3 x_{2} \leq 12 \\
& 6 x_{1}+8 x_{2} \leq 24
\end{aligned}
$$

$$
\text { and } x_{1}, x_{2} \geq 0 \text { are integers. }
$$

9. a) Use two-phase simplex method to solve

Maximize $Z=5 x_{1}+3 x_{2}$
subject to $3 x_{1}+5 x_{2} \leq 15$

$$
\begin{gather*}
5 x_{1}+2 x_{2} \leq 10 \\
x_{1}, x_{2} \geq 0 \tag{7}
\end{gather*}
$$

b) Solve the game whose pay-off matrix is given below :

Player $B$

|  | $B_{1}$ | $B_{2}$ | $B_{3}$ | $B_{4}$ |
| :---: | :---: | :---: | :---: | :---: |
| $A_{1}$ | 3 | 2 | 4 | 0 |
| $A_{2}$ | 3 | 4 | 2 | 4 |
| $A_{3}$ | 4 | 2 | 4 | 0 |
| $A_{4}$ | 0 | 4 | 0 | 8 |


10. a) A hospital has the following requirement ofnurses :

| Period | Clock Time (24 hour day) | Minimum requirement |
| :---: | :---: | :---: |
| 1. | $6 \mathrm{am}-10 \mathrm{am}$ | 60 |
| 2. | $10 \mathrm{am}-2 \mathrm{pm}$ | 70 |
| 3. | $2 \mathrm{pm}-6 \mathrm{pm}$ | 60 |
| 4. | $6 \mathrm{pm}-10 \mathrm{pm}$ | 50 |
| 5. | $10 \mathrm{pm}-2 \mathrm{am}$ | 20 |
| 6. | $2 \mathrm{am}-6 \mathrm{am}$ | 30 |

Nurses report to the hospital at the beginning of each period and work for eight consecutive hours. The Hospital wants to determine the minimum number of nurses so that there may be sufficient number of nurses available for each period. Formulate this as an L.P.P. 7
b) Find the critical path and the minimum time to complete the project of the following graph :

11. a) In Birth and Death model show that the expected number of customers in the system is $\frac{\rho}{(1-\rho)}$, where $\rho=\frac{\lambda}{\mu}, \lambda=$ mean arrival rate of customer, $\mu=$ mean service rate of the customer.

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b) The time estimate in hours for the activities of a PERT network are given below :

| Activity $(i-j)$ | $1-2$ | $1-3$ | $1-4$ | $2-5$ | $3-5$ | $4-6$ | $5-6$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Optimistic Time | 1 | 3 | 6 | 7 | 8 | 9 | 11 |
| Most likely time | 2 | 4 | 7 | 8 | 9 | 10 | 12 |
| Permissible time | 3 | 5 | 8 | 9 | 10 | 11 | 13 |

i) Draw a project network.
ii) Determine the expected project length.
iii) Calculate the standard deviation and variance of the project length.

