PH-301

PHYSICS-II

Time Allotted: 3 Hours   Full Marks: 70

The questions are of equal value.
The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

GROUP A
(Multiple Choice Type Questions)

1. Answer any *ten* questions. $10 \times 1 = 10$

(i) Condition of coplanarity of three vectors $\bar{\alpha}, \bar{\beta}$ and $\bar{\gamma}$ is

(A) $\bar{\alpha}.(\bar{\beta} + \bar{\gamma}) = 0$
(B) $\bar{\alpha}.(\bar{\beta} \times \bar{\gamma}) = 0$
(C) $\bar{\alpha} \times (\bar{\beta} + \bar{\gamma}) = 0$
(D) $\bar{\alpha}.(\bar{\beta}.\bar{\gamma}) = 0$

(ii) In free space, Poisson's equation is

(A) $\nabla^2 V = -\dfrac{\rho}{\varepsilon_0}$
(B) $\nabla^2 V = \dfrac{\rho}{\varepsilon_0}$
(C) $\nabla^2 V = 0$
(D) $\nabla^2 V = \rho \varepsilon_0$

(iii) The unit of $\iint \vec{D}.\vec{ds}$ is

(A) Coulomb
(B) Farad
(C) Watt
(D) Volt

(iv) The electronic polarisability for a rare gas atom is

(A) $\alpha_e = \dfrac{\varepsilon_r - 1}{\varepsilon_0 N}$
(B) $\alpha_e = \varepsilon_0 \dfrac{(\varepsilon_r - 1)}{N}$
(C) $\alpha_e = N(\varepsilon_r - 1)$
(D) $\alpha_e = \dfrac{N}{(\varepsilon_r - 1)}$

(v) Magnetic field due to an infinitely long solenoid of radius a and having n no. of turns per unit length, carrying current I is

(A) $\dfrac{\mu_0}{4\pi} \dfrac{2\pi I}{a}$
(B) $\dfrac{\mu_0}{4\pi} \dfrac{2I}{a}$
(C) $\dfrac{1}{4\pi\mu_0} \dfrac{I}{a}$
(D) zero

(vi) The divergence of magnetic flux density is

(A) 0
(B) 1
(C) − 1
(D) $\propto$

(vii) The equation of continuity in a steady charge distribution is

(A) $\vec{\nabla}.\vec{J} = 0$
(B) $\vec{\nabla} \times \vec{J} = 0$
(C) $\vec{\nabla} \times \vec{J} = \rho$
(D) $\vec{\nabla}.\vec{J} = \rho$

(viii) Poynting vector is represented by

(A) $\vec{P} = \dfrac{1}{2\epsilon}\left(\vec{E} \times \vec{H}\right)$
(B) $\vec{P} = \dfrac{1}{2\mu}\left(\vec{E} \times \vec{H}^*\right)$
(C) $\vec{P} = \sqrt{\dfrac{\mu}{\epsilon}}\left(\vec{E} \times \vec{H}\right)$
(D) $\vec{P} = \dfrac{1}{2}\left(\vec{E} \times \vec{H}^*\right)$

(ix) For T. > 0 K, the probability of occupancy of an electron at Fermi energy level is

(A) $\dfrac{1}{2}$
(B) 0
(C) 1
(D) $\dfrac{1}{4}$

(x) Which of the following function are eigen function of the operator $\dfrac{d^2}{dx^2}$

(A) $\psi = c \ln x$
(B) $\psi = cx^2$
(C) $\psi = \dfrac{c}{x}$
(D) $\psi = c\, e^{-mx}$

(xi) For a particle trapped in a box of length l, the average value of momentum <P> is

(A) $\dfrac{h}{al}$

(B) 0

(C) 1

(D) $\dfrac{h^2}{al}$

(xii) If $\psi(x,t)$ is a normalised one dimensional wave function, then-

(A) $\int_{-a}^{+a} \psi^* \psi \, dx = 1$

(B) $\int_{-a}^{+a} \psi^* \psi \, dx = \dfrac{1}{2}$

(C) $\int_{-a}^{+a} \psi^* \psi \, dx = 0$

(D) $\int_{-a}^{+a} \psi^* \psi \, dx = \alpha$

(xiii) The ground state energy of a particle moving in a one dimensional potential box is given in terms of length L of the box by

(A) $\dfrac{2\hbar^2}{8mL^2}$

(B) $\dfrac{\hbar^2}{8mL^2}$

(C) $\dfrac{h^2}{8mL^2}$

(D) 0

(xiv) A system is called strongly degenerate if

(A) $\dfrac{N_1}{g_1} = 1$

(B) $\dfrac{N_1}{g_1} \gg 1$

(C) $\dfrac{N_1}{g_1} \ll 1$

(D) $g_1 = 1$

(xv) Average energy $\langle E \rangle$ of electron in a metal at $T = 0K$ is

(A) E_F

(B) $\dfrac{E_F}{2}$

(C) $\dfrac{3}{5} E_F$

(D) $\dfrac{3}{2} E_F$

GROUP B
(Short Answer Type Questions)

Answer any *three* questions.   3 × 5 = 15

2. (a) Use Gauss' law to calculate the electric field between infinite extent parallel plate capacitor carrying charge density σ and mutual separation d.   2+3

   (b) Verify whether the potential function $V(x, y)$ satisfy Laplace's equation or not. Find also the density.

3. (a) Derive Poisson's equation and Laplace's equation from fundamentals.   3+2

   (b) Show that the function $V = 2x^2 + 7y - 2z^2$ represents the potential function in a charge-free region.

4. (a) Write Lorentz force equation and explain the symbols used.   2+3

   (b) Deduce an expression of the force experienced by a current element placed in a magnetic field.

5. A particle of mass m is confined by infinite potentials within $x = 0$ to $x = L$.   1+4

   (a) Write the Schrödinger's equation to describe the motion of the particle.

   (b) Solve the equation to find out the normalized eigen functions.

6. Write the basic assumptions in deriving the (i) MB (ii) BE and (iii) FD statistics. How does the distribution function differ in the three cases?   5

GROUP C
(Long Answer Type Questions)

Answer any *three* questions.   3 × 15 = 45

7. (a) Show that the potential function $V = V_0 (x^2 - 2y^2 + z^2)$ satisfies Laplace's equation where V_0 is a constant.   3+4

   (b) A very long cylindrical object carries charge distribution proportional to the distance from the axis (r). If the cylinder is of radius a, then find the electric field both at $r > a$ and $r < a$ by the application of Gauss' law in electrostatics.   1+1

   (c) Define conductivity and mobility of conducting material. Define current density. Derive the equation of continuity for steady current flow.   2+4

8. (a) Write down Maxwell's equations in electromagnetic field and explain physical significance of four equations.   2+2

  (b) What do you mean by Maxwell correction of Amperes law?   3

  (c) Write down Maxwell's equations in free space. Show that electric field ($\vec{E}$) and magnetic field ($\vec{B}$) and propagation vector are mutually perpendicular.   2+4

  (d) If $\vec{E} = \sin(y - t)\hat{k}$ and $\vec{B} = \sin(y - t)\hat{i}$ then show that these constitute an electromagnetic field.   2

9.   Using the wave function of a quantum mechanical particle   (3+3)+4+

    $\psi(x) = Ae^{i(kx - \omega t)}$ where all symbols have their usual meanings-   (3+2)

  (a) (i) deduce the operator form of momentum

         Or

    (ii) deduce the operator form of energy.

  (b) Check whether position and momentum of a quantum mechanical particle are commutative or not.

  (c) Find the possible arrangements of 3 particles A, B and C in 3 cells according to the three statistics.

  (d) Obtain the equation of motion of a simple pendulum from its Lagrangian representation. Construct the Hamiltonian.

10. (a) What do you mean by cyclic coordinate? Explain with an example.   2

  (b) Show that if generalized force for a conservative system is zero then the generalize momentum will be conserved.   3

  (c) The wave function in a one-dimensional potential box with rigid walls is given by   3+3

    $\psi(x) = \sqrt{\dfrac{2}{l}} \sin\dfrac{n\pi x}{l}, |x| \le l,$

        = 0 otherwise

    Find the expectation value of $x^{\wedge}$ and $p^{\wedge}$.

  (d) Consider $\varphi = c_1 \varphi_1 + c_2 \varphi_2$, where φ_1 and φ_2 are orthonormal energy eigenstates of a system corresponding to energy E_1 and E_2 at $t = 0$. If φ is normalized and   1+2+1

    $c_1 = \dfrac{1}{\sqrt{3}}$ then what is the value of c_2? Find the expectation value of E_2. Write the wave function at a subsequent time.

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11. (a) Use Lagrange's equation of motion to determine the motion of a mass, m, sliding without friction down an inclined plane of angle, α.   5

  (b) Write down the basic postulates of BE statistics. Define fermions.   2+1

  (c) Obtain the expression for the eigen function of the momentum operator   4

    $\hat{p}_x = -i\hbar\dfrac{d}{dx}$ corresponding to an eigen value p_x.

  (d) Prove that generalized momentum (p_j) corresponding to cyclic co-ordinate (p_j) is a constant of motion.   3

12. (a) Explain graphically the Fermi distribution at zero degree absolute and non-zero degree absolute temperature.   3

  (b) In a system of two particles, each particle can be in any one of three possible quantum states. Find the ratio of the probability that the two particles occupy the same state to the probability that the two particles occupy different states for MB, BE and FD statistics.   6

  (c) Calculate the Fermi temperature and Fermi velocity for sodium whose Fermi level is 1.6 eV. (Given, Boltzmann constant = 1.38×10^{-23} J(K)$^{-1}$; mass of electron = 9.1×10^{-31} Kg).   3

  (d) Illustrate microstates and macro states with suitable examples. Elucidate the concept of energy levels and energy states.   2+1