

CS/B.Tech/CSE/IT/Even/Sem-4th/M-401/2015



WEST BENGAL UNIVERSITY OF TECHNOLOGY

M-401

MATHEMATICS-III

Time Allotted: 3 Hours

Full Marks: 70

The questions are of equal value.

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP A

(Multiple Choice Type Questions)

1. Answer any *ten* questions. 10×1 = 10
- (i) If A and B are two events with $P(A) = 0.4$, $P(B) = 0.3$ and $P(A \cap B) = 0.2$, then $P(A^c \cap B)$ is
 (A) 0.1 (B) 0.2 (C) 0.3 (D) 0.4
- (ii) If two events A and B are independent, then
 (A) $P(A \cap B) = P(A)P(B)$ (B) $P(A+B) = P(A)+P(B)$
 (C) $P(A-B) = P(A)P(B)$ (D) $P(A \cap B) = P(A)P(B/A)$
- (iii) A fair die is thrown. The probability that either an odd number or a number greater than 4 will turn up is
 (A) $\frac{2}{5}$ (B) $\frac{3}{7}$ (C) $\frac{2}{7}$ (D) $\frac{2}{3}$
- (iv) The mean of a uniform distribution with parameters a and b is
 (A) $b - a$ (B) $b + a$ (C) $\frac{(a+b)}{2}$ (D) $\frac{b-a}{2}$
- (v) The variance of a random variable X is
 (A) $[E(X)]^2$ (B) $E(X^2)$
 (C) $E(X^2) - [E(X)]^2$ (D) $E(X^2) - [E(X)]$

CS/B.Tech/CSE/IT/Even/Sem-4th/M-401/2015

(vi) A random variable x has the following pdf

$$f(x) = k \quad -2 < x < 2$$

$$= 0 \quad \text{otherwise}$$

Then the value of the constant k is

- (A) $\frac{1}{8}$ (B) $\frac{1}{2}$ (C) $\frac{1}{4}$ (D) $\frac{1}{12}$

(vii) The number of generators of an infinite cyclic group is

- (A) 1 (B) 2 (C) infinite (D) none of these

(viii) If R is a ring without zero divisors, then $x \cdot y = 0$ implies

- (A) $x = 0$ or $y = 0$ (B) $x = 0$ and $y = 0$
 (C) $x = 0, y \neq 0$ (D) $x \neq 0, y = 0$

(ix) The order of the dihedral group D_4 is

- (A) 4 (B) 6 (C) 8 (D) 64

(x) Let X be a Poisson Random Variate and $E(X) = \lambda$. Then $E[(X+1)^2]$ will be

- (A) λ (B) $\lambda^2 + 2\lambda$
 (C) $\lambda^2 + 2\lambda + 1$ (D) $\lambda^2 + 3\lambda + 1$

(xi) A complete graph is called Kuratowski's first graph if it has

- (A) 5 vertices (B) 4 vertices (C) 6 vertices (D) 7 vertices

(xii) Let C_{97} be a circuit with 97 vertices. Then $\chi(C_{97})$ is equal to

- (A) 97 (B) 98 (C) 2 (D) 3

(xiii) The maximum number of edges in a simple connected graph with n vertices is

- (A) $2 \cdot {}^n C_2$ (B) ${}^n C_2$ (C) $(n-1)$ (D) n^2

(xiv) If H is a subgroup of a group G and a, b are two distinct elements of G , then indicate which of the following statements is true

- (A) $aH = Ha$ (B) $Ha \cap Hb = \phi$
 (C) $Ha \cap Hb \neq \phi$ and $Ha \neq Hb$ (D) $aH = bH$

(xv) A null hypothesis is a statistical hypothesis which is setup and whose validity is tested for possible

- (A) acceptance (B) rejection (C) testing (D) none of these

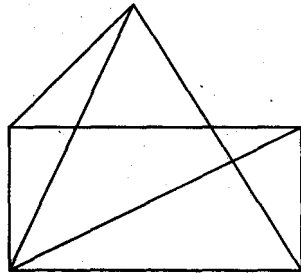
GROUP B**(Short Answer Type Questions)**Answer any *three* questions.

3×5 = 15

2. Show that for any two subgroups H and K of a group G , $H \cap K$ is also a subgroup of G .

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3. If in a ring R with unity, $(xy)^2 = x^2y^2$ for all $x, y \in R$, then show that R is commutative.
4. A random sample of size 20 from a normal population gives the sample mean of 42 and the sample standard deviation of 6. Test the hypothesis that the population mean is 44. Value of t distribution with 19 degrees of freedom at 5% level is 2.09.
5. Urn I has 2 white and 3 black balls. Urn II has 4 white and 1 black and Urn III has 3 white and 4 black balls. An urn is selected at random and ball drawn at random is found to be white. Find the probability that Urn I was selected.
6. Draw the dual of the graph.



GROUP C
(Long Answer Type Questions)

Answer any *three* questions.

3×15 = 45

7. (a) State and prove Euler's formula for a connected planar graph. 5
- (b) If G is a group and H is a subgroup of index 2 in G , then prove that H is a normal subgroup in G . 5
- (c) A box contains 5 defective and 10 non defective lamps. 8 lamps are drawn at random in succession without replacement. What is the probability that the 8th lamp drawn is the 5th defective one? 5
8. (a) Show that the set G of all ordered pairs (a, b) with $a \neq 0$, of real numbers a, b forms a group with respect to ' \circ ' defined by $(a, b) \circ (c, d) = (ac, bc + d)$. 5
- (b) Define Normal distribution and find its mean, variance and standard deviation. 6
- (c) It is seen that a cricketer becomes out within 10 runs in 3 out of 10 innings. If he plays 4 innings, what is the probability that he will becomes (i) out twice (ii) out at least once within 10 runs. 4

CS/B.Tech/CSE/IT/Even/Sem-4th/M-401/2015

9. (a) Let G be a simple connected planar graph with n vertices, e edges and f faces. Prove that the following inequalities must hold 5
- (a) $n \geq \frac{3}{2}f$;
 (b) $e \leq (3n - 6)$.
- (b) A normal population has mean 0.1 and standard deviation 2.1. Find the probability that the mean of a sample of size 900 will be negative. (Given that $P(|z| < 1.43) = 0.847$) 5
- (c) For any two integers m, n define $m \oplus n = m + n - 1$ and $m \odot n = m + n - mn$. Prove that the set of integers Z forms a commutative ring with identity with these two binary operations. 5
- 10.(a) If a population has normal distribution with parameter μ and σ , then show that the statistic $\hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \mu)^2$ is a maximum likelihood estimator of σ^2 when μ is known. 5
- (b) A machine part was designed to withstand an average pressure of 120 units. A random sample of size 100 from a large batch was tested and it was found that the average pressure which these parts can withstand is 105 units with a standard deviation of 20 units. Test at 5% level of significance whether the batch meet the specification. Suppose the population has normal distribution and given that $\phi(1.645) = 0.45$. 5
- (c) Let G be a group and let $a \in G$. Prove that the mapping $\phi_a : G \rightarrow G$ defined by $\phi_a(x) = a \cdot x \cdot a^{-1}$ is an isomorphism on G . 5
- 11.(a) Show that a field does not contain any zero divisor. 4
- (b) Survey of 320 families with 5 children each revealed the following distribution: 6
- | | | | | | | |
|----------------|----|----|-----|----|----|----|
| No. of boys: | 5 | 4 | 3 | 2 | 1 | 0 |
| No. of girls: | 0 | 1 | 2 | 3 | 4 | 5 |
| No. of family: | 14 | 56 | 110 | 88 | 40 | 12 |
- Is the result consistent with the hypothesis that male and female births are equally probable? The 5% value of X^2 with 5 d.o.f is 11.07.
- (c) A random variable X follows uniform distribution with parameters 0 and 1. Find the pdf of the random variable $U = \sqrt{X}$. 5