CS/B.Tech(New)/CSE/IT/Even/4th Sem/M-401/2014

2014
Mathematics - III

Time Alloted : 3 Hours   **Full Marks : 70**

The figure in the margin indicate full marks.
Candidates are required to give their answers in their
own words as far as practicable

GROUP - A
(Multiple Choice Type Questions)

1.   Choose the correct alternatives for any ten of the following :

$10 \times 1 = 10$

i)   A problem in Mathematics is given to three students A, B and C. The chances of solving the problem by A, B and C are $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$ respectively. The probability that the problem will be solved is

a) $\frac{2}{5}$ b) $\frac{3}{5}$

c) $\frac{1}{60}$ d) $\frac{47}{60}$

ii)   Let G be a Group and $a, b \in G$. Then $\left(a^{-1}b\right)$ is equal to

a) ab^{-1} b) b^{-1}a

c) a^{-1}b^{-1} d) b^{-1}a^{-1}

iii)   If a simple graph has 15 edges then sum of the degrees of

1184   1   [Turn over]

all the vertices is

      a) 25   b) 24

      c) 50   d) 30

iv)   The probability that a leap year selected at random will contain 53 Sundays is

      a) 2/53   b) 52/53

      c) 1/7   d) 2/7

v)   The mean and variance of a distribution is given to be 10 and 6 respectively. Then the distribution is

      a) Standard Normal Distribution

      b) Binomial Distribution

      c) Poisson Distribution

      d) None of these

vi)   A random variable X has the following probability density function:

$$f(x) = \begin{cases} kx, 0 \le x \le 1 \\ 0, otherwise \end{cases}$$

    The value of k is

      a) 1   b) 2

      c) 4   d) none of these

vii)   The statistic t is said to be unbiased estimator of a population parameter θ when

      a) $E(t) = \theta$ b) $E(t^2) = \theta$

      c) $E(t^2) = [E(\theta)]^2$ d) $[E(t)]^2 = [E(\theta)]^2$

viii)   The number of unit elements of the ring (Z, +,..)

      a) 2   b) 3

      c) 1   d) infinite.

ix)   Chromatic number of a complete graph with 15 vertices is

      a) 12   b) 13

      c) 14   d) 15

x)   In a Poisson distribution if $2P(x = 1) = P(x = 2)$, then the variance is

a) 0   b) -1

c) 4   d) 2

xi) Let A and B be two events with $P(A) = 1/2$, $P(B) = 1/3$, $P(A \cap B) = 1/4$. Then $P(A/B)$ is

       a) 3/4   b) 5/4

       c) 7/4   d) 2.

xii) In 'Goodness of fit' which of the following is used as test statistic

       a) normal variate   b) t variate

       c) Poisson variate   d) X^2 variate.

xiii) Let s be a finite set containing n elements. Then the probability that a mapping $f : s \rightarrow s$ will be a birective mapping is

       a) $\dfrac{n^o}{n!}$ b) $\dfrac{n!}{n^n}$

       c) $\dfrac{n-1}{n!}$ d) $\dfrac{n+1}{n!}$

xiv) If G is a non-planar graph, then the number of vertices of G is

       a) 2   b) 3

       c) 4   d) 6

xv) Which one of the following is not a cyclic group.

       a) $(Z, +)$ b) $(Z_4, +)$

       c) $(Q, +)$ d) $(Z_{15}, +)$

GROUP - B
(Short Answer Type Questions)
Answer any three of the following.

$3 \times 5 = 15$

2. Prove that a group $(G, *)$ is commutative if and only if $(a * b)^2 = a^2 * b^2$, for all $a, b \in G$,

3. If T is an unbiased estimator of θ, then show that T^2 is a biased estimator of θ^2.

4. In a certain city, the daily consumption of electric power (in millions of kilowatt hours) is a random variable having the probability density

$$f(x) = \frac{1}{9} x e^{-x/3}, x > 0$$

$$= 0, \qquad , x \leq 0$$

If the city's power plant has a daily capacity of 12 million kilowatt-hours, what is the probability that this power supply will be inadequate on any given day?

5. Show that the 7th roots of unity form a cyclic group. Find all the generators of this group.

6. Find the dual of the following graph:
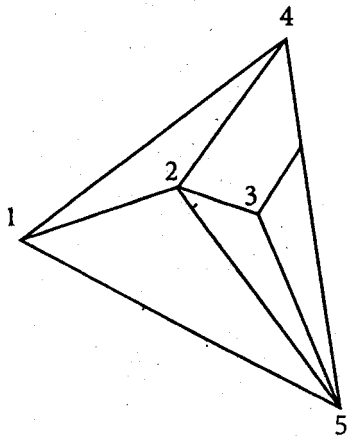
GROUP - C
(Long Answer Type Questions)
Answer any three of the following.

3 x 15 = 45

7. a) If G be a connected planar graph with n vertices, e number of edges and f number of faces, prove that n-e+f=2.

b) Suppose that an airplane engine will fall, When in flight, with probability (1-p) independently from engine to engine; Suppose that the airplane will make a successful flight if at least 50% of its engines remain operative. For what values of p is a four-engine plane preferable to a two-engine plane?

c) Find the mean of an uniform distribution.   (7 + 6 + 2 =15)

8. a) Prove that a graph with *n* vertices is a tree if and only if its chromatic polynomial $p_n(\lambda) = \lambda(\lambda-1)^{n-1}$.

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b) If the weekly wage of 10,000 workers in a factory follows normal distribution with mean and standard deviation Rs. 70 and Rs. 5 respectively, then find the expected number of workers whose weekly wages are

5

i) between Rs. 66 and Rs. 72

ii) less than Rs 66.

[Given that the area under the standard normal curve between z = 0 and z = 0,4 is 0.1554 and z = 0 and z = 0.8 is 0.2881].

c) Prove that the order of each subgroup of a finite group is a divisor of the order of the group.

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9. a) Give an example to show that a graph is drawn in two different ways as planer graph, but its dual are non isomorphic.   5

    b) Prove that every group of prime order is cyclic.
    5

    c) Let $GL(2,\mathbb{R})$ denote the set of all non singular 2 x 2 matrices with real entries. Show that

    $$SL(2,\mathbb{R}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in Gl(2,\mathbb{R}) : ad - bc = 1 \right\}$$

    is a normal subgroup of $GL(2,\mathbb{R})$.   5

10. a) The probability density of a random variable z is given by

    $$f(z) = \begin{cases} kze^{-z^2}, & for\ z > 0 \\ 0, & for\ z \leq 0 \end{cases}$$

    Find the value of k and find out the corresponding distribution function of z.
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    b) A random sample of size n=100 is taken from an infinite population with the mean $\mu = 75$ and the variance $\sigma^2 = 256$. Based on Chebyshev's theorem with what probability can we assert that the value we obtain for $\overline{X}$ will fall between 67 and 83. 5

    c) Prove that every finite integral domain is a field.   5

11. a) Show that there does not exist any isomorphism from the group $(\mathbb{R},+)$ to group $(\mathbb{R}^*,\cdot)$. ($\mathbb{R}$ is the set of all real numbers and $\mathbb{R}^*$ is the set of all nonzero real numbers)   5

    b) Suppose that 100 tires made by a certain manufacturer lasted on the average 21819 miles with a standard deviation of 1295 miles. Test the null hypothesis μ = 22000 miles against the alternate

hypothesis $\mu < 22000$ miles at the 0.05 level of significance.   5

c) Let $X_1, X_2, \ldots, X_n$ be the values of a random sample from an exponential population that is $f(x_i) = \frac{1}{\theta} e^{-\frac{x}{\theta}}$ for $X_i > 0$. Then find the maximum likelihood estimator of its parameter θ.   5