

CS/B.Tech(New)/CSE/IT/SEM-4/M-401/2013

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iii) If t is a statistic such as $E\{t^2\} = 5$ and $E\{t\} = 2$ then the standard error of t is

- a) 0
- b) 1
- c) 2
- d) none of these.

iv) If the exponential distribution is given by the probability density function $f(x) = e^{-x}$, $0 < x < \infty$, then the mean of the distribution is

- a) 1
- b) 3
- c) $\frac{1}{3}$
- d) none of these.

v) The probability of an event A is $\frac{1}{3}$, that of $A + B$ is $\frac{1}{2}$ and that of AB is $\frac{1}{4}$. Then the probability of B is

- a) $\frac{1}{12}$
- b) $\frac{5}{12}$
- c) $\frac{1}{6}$
- d) none of these.

vi) Which one of the following sets forms a group under usual multiplication of complex numbers?

- a) $\{1, i\}$
- b) $\{1, \omega, \omega^2\}$
- c) $\{1, \omega^2\}$
- d) $\{1, \omega\}$.

vii) The distribution for which the mean and variance are equal is

- a) Poisson
- b) normal
- c) binomial
- d) exponential.

viii) In a Binomial (n, p) distribution, if its mean and variance are 2 and $\frac{4}{3}$ respectively, then the values of n and p are

- a) $8, \frac{1}{4}$
- b) $6, \frac{1}{3}$
- c) $4, \frac{1}{2}$
- d) none of these.

ix) If G is a connected planar graph with n vertices, e edges and f faces, then $n - e + f = 2$. This statement is

- a) True
- b) False.

x) The mean of Binomial variate is

- a) np
- b) $np(1-p)$
- c) \sqrt{np}
- d) none of these.

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- xi) Kuratowski's graph is a
- a) planar graph b) regular graph
- c) tree d) none of these.

xii) The order of the dihedral group D_4 is

- a) 4 b) 6
- c) 8 d) 64.

xiii) Every finite integral domain is a field. This statement is

- a) True b) False.

xiv) If A and B are two subgroups of a group G , then which of the following is always a subgroup of G ?

- a) $A \cup B$ b) $G - A$
- c) $G - B$ d) $A \cap B$.

xv) The symmetric group S_3 has

- a) 6 elements b) 8 elements
- c) 9 elements d) none of these.

GROUP - B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

Let $(\mathbb{Q}, +)$ be the additive group of rational numbers and (\mathbb{Q}^+, \cdot) be the multiplicative group of positive rational numbers. Are these two groups isomorphic? Justify your answer.

Prove Baye's theorem for repeated trials.

Examine whether function $|x|$ in $(-1, 1)$ and zero elsewhere is a density function.

Show that a connected graph is Eulerian if and only if each of its vertices is of even degree.

Show that a field does not contain any zero divisor.

GROUP - C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

a) Prove that a planar graph with n vertices, e number of edges and k number of components determines f number of regions, where $f = e - n + k + 1$. 8

b) Let \bar{X} be the sample mean of samples of size n drawn at random from a population which is normally distributed with mean μ and variance σ^2 . Find the standard error of the statistic \bar{X} . 7

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8. a) Find the mathematical expectation of the number points obtained in a single throw of an unbiased die.
- b) Define Poisson distribution and find its mean & variance.
- c) Let f be a ring homomorphism from the ring \mathbb{Z} of integers into itself such that $f(1) = 1$. Determine homomorphism f .
9. a) Show that any simple connected planar graph with n vertices ($n \geq 3$) has at most $(3n - 6)$ edges.
- b) Prove that every nontrivial subgroup of the additive group \mathbb{Z} of integers is cyclic.
- c) Let R and S be two rings and $f: R \rightarrow S$ be a ring homomorphism. Show that kernel of f is a subring of R .
10. a) Determine the mean and variance of exponential distribution.
- b) Show that every cyclic group is commutative.
- c) Let H be a normal subgroup of a group G and G/H the set of all cosets of H in G . Show that G/H forms a group under the composition

$$(aH) \cdot (bH) = (ab)H \text{ for all } a, b \in G.$$

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- a) The probability density function of a random variable X is assumed to be of the form $f(x) = cx^\alpha$, $0 \leq x \leq 1$ for some number and constant c . If $\{X_1, X_2, \dots, X_n\}$ is a random sample of size n , find the maximum likelihood estimate of α . 5
- b) Let S' be the set defined by $S' = \{z \in \mathbb{C} : |z| = 1\}$, where \mathbb{C} is the set of all complex numbers. Show that S' forms a commutative group under usual multiplication of complex numbers. 5
- c) Let R be the additive group of real numbers and \mathbb{C}^* be the multiplicative group of nonzero complex numbers. If $f: R \rightarrow \mathbb{C}^*$ is a group homomorphism defined by $f(x) = e^{2\pi i x}$ for all $x \in R$, find the kernel of f . 5