

Name : .....

Roll No. : .....

Invigilator's Signature : .....

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**2010-11**

**MATHEMATICS**

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words  
as far as practicable.

**GROUP - A**

**( Multiple Choice Type Questions )**

1. Choose the correct alternatives for any ten of the following :

$$10 \times 1 = 10$$

i) If  $F[f(x)] = F(s)$  represents the Fourier transform of the function  $f(x)$ , then  $F[f(ax)]$  (' $a$ ' being a constant) equals

a)  $F(s/a)$       b)  $a F(s)$

c)  $(1/|a|)F(s/a)$       d)  $(1/a^2)F(as)$ .

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- ii) A function  $f(x)$ ,  $a < x < b$ , can be expanded in a Fourier series
- only if it is continuous everywhere
  - even if it is discontinuous at a finite number of points in  $(a, b)$
  - even if it is unbounded in  $(a, b)$
  - only if it is both continuous & bounded in  $(a, b)$ .
- iii) Three unbiased coins are tossed simultaneously. This is repeated four times. Then the probability of getting at least one head each time is
- $(1/8)^4$
  - $(2/8)^4$
  - $(7/8)^4$
  - $(3/8)^4$ .
- iv) For a Poisson distribution  $P(X)$  is  $P(1) = P(2)$ , then  $P(0)$  is
- $1/e$
  - $1/e^2$
  - $1/e^3$
  - none of these.
- v) A graph has 10 vertices and 15 edges. Its circuit rank is
- 25
  - 12
  - 6
  - 5.

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- vi) A binary tree has 11 vertices. The minimum and maximum height of the tree is



- vii) If  $f(x)$  is an odd function then  $\mathcal{F}(f(x))$  is given by

- a)  $F(s) = 2F_s(s)$       b)  $F(s) = 2iF_s(s)$   
 c)  $F(s) = 0 \cdot 5iF_s(s)$       d)  $2F(s) = iF_s(s)$ ,

where  $\mathcal{F}$  denotes Fourier Transform.

- viii) The order of pole  $z = 0$  of the function  $\frac{\cos z}{z^3}$  is



- ix) If  $X$  is normally distributed with zero mean and unit variance, then the expectation of  $X^2$ , is



- x) The maximum and minimum values for correlation coefficient are

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- xi) If a simple graph has 15 edges then sum of the degrees of all the vertices is



- xii) A closed walk in which no vertex (except its terminal vertices) appear more than once is called



**GROUP - B**

**( Short Answer Type Questions )**

**Answer any three of the following**

$$3 \times 5 = 15$$

2. If  $f(z) = \frac{xy^2(x+iy)}{x^2+y^4}$ ,  $z \neq 0$  &  $f(0) = 0$ , then prove that

$\frac{f(z) - f(0)}{z} \rightarrow 0$  as  $z \rightarrow 0$  along any radius vector but not as

$z \rightarrow 0$  in any manner.

3. If  $f$  is analytic function then show that  $\nabla^2 |f(z)|^2 = 4 \frac{\partial(u, v)}{\partial(x, y)}$

where  $f(z) = u + iv$  and  $z = x + iy$ .

4. Expand the following function in a Fourier series in  $[-\pi, \pi]$

$$f(x) = \begin{cases} -\frac{1}{2}(\pi + x) & \text{when } -\pi \leq x < 0 \\ \frac{1}{2}(\pi - x) & \text{when } 0 \leq x < \pi \end{cases}$$

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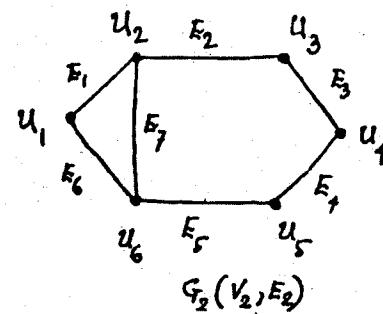
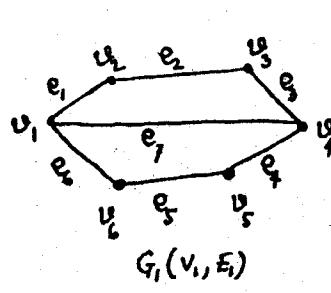
5. Show that  $f(x)$  given by

$$f(x) = \begin{cases} x & \text{for } 0 < x < 1 \\ k - x & \text{for } 1 < x < 2 \\ 0 & \text{elsewhere} \end{cases}$$

is a probability density

function for a suitable value of k. Calculate the probability that the random variable lies between  $1/2$  and  $3/2$ .

6. Define isomorphism of two graphs. Show whether the following graphs are isomorphic or not :



### GROUP - C

#### ( Long Answer Type Questions )

Answer any three of the following.  $3 \times 15 = 45$

7. a) Consider Heavyside unit function

$$h(1-|t|) = 0, |t| > 1$$

$$= 1, |t| \leq 1$$

Prove that  $F^{-1}(\sin s/s) = h(1-|x|)$  where  $F^{-1}$  is the inverse Fourier transform i.e.,  $F^{-1}(F(s)) = f(t)$ .

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- b) Using Parseval's identity of Fourier transform prove that

$$\int_0^{\infty} (1 - \cos x/x)^2 dx = \pi/2$$

- c) Using Fourier transform solve the heat equation

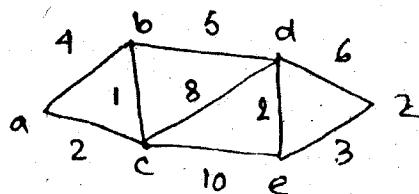
$$\partial^2 u / \partial x^2 = (1/c^2)(\partial u / \partial t), -\infty < x < \infty, t > 0$$

with boundary condition  $u(x, t) \rightarrow 0, \partial u(x, t) / \partial x \rightarrow 0$

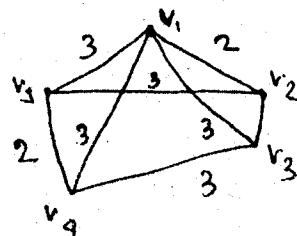
as  $|x| \rightarrow \infty$  & initial condition  $u(x, 0) = e^{-x^2/4c^2}, -\infty < x < \infty$

3 + 4 + 8

8. a) Using Dijkstra's algorithm find the length of the shortest path of the following graph :



- b) Find by Prim's Algorithm a minimum spanning tree from the following graph :



8 + 7

9. a) Solve the differential equation :

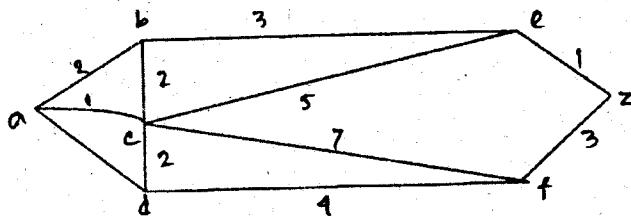
$$k \partial^2 u / \partial x^2 = \partial u / \partial t, -\infty < x < \infty, t > 0$$

with  $u(x, t) = 0$  as  $x \rightarrow \pm\infty$ ,  $\partial u / \partial t = 0$  as  $x \rightarrow \pm\infty$  and

$$u(x, 0) = f(x), -\infty < x < \infty.$$

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- b) Apply Dijkstra's algorithm to determine a shortest path between  $a$  to  $z$  in the following graph.



10. a) The probability density function of a random variable  $X$  is  $f(x) = K(x - 1)(2 - x)$ , for  $1 \leq x \leq 2$ .  
= 0, otherwise.

Determine –

- (i) the value of the constant  $k$  and

$$(ii) P\left(\frac{5}{4} \leq X \leq \frac{3}{2}\right).$$

- b) In a normal distribution, 31% of the items are under 45 and 8% are above 64. Find the mean and standard deviation. [Given that  $P(0 < Z < 1.405) = 0.42$  and  $P(-0.496 < Z < 0) = 0.19$ ]

- c) If the equations of two Regression lines obtained in a correlation analysis are  $3x + 12y - 19 = 0$  and  $9x + 3y = 46$ . Determine which one is Regression equation of  $y$  on  $x$  and which one is the regression equation of  $x$  on  $y$ . Find the means of  $x$  on  $y$  and correlation coefficient between  $x$  and  $y$ . 4 + 5 + 6

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11. a) If  $f(x) = \begin{cases} 0 & -\pi \leq x \leq 0 \\ \sin x & 0 \leq x \leq \pi \end{cases}$ , prove that

$$f(x) = \frac{1}{\pi} + \frac{1}{2} \sin x - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\cos 2nx}{4n^2 - 1}$$

Hence show that

$$\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \dots = \frac{1}{2}$$

- b) Evaluate  $\int_C \frac{4 - 3z}{(z - 1)z(z - 3)} dz$ , where C is the circle  $|z| = \frac{5}{2}$ .
- c) Show that  $u(x, y) = x^3 - 3xy^2$  is harmonic in C and find a function  $v(x, y)$  such that  $f(z) = u + iv$  is analytic.

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