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Paper Code : M-201

MATHEMATICS-II

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) The value of the integral $\int_{-1}^{1} \frac{1}{x^2}\,dx$ is

a) -2 b) -1

c) does not exist d) 0.

ii) The maximum number of edges in a simple graph with 10 vertices is

a) 45 b) 20

c) 10 d) 9.

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iii) The general solution of $y = px + \sqrt{a^2p^2 + b^2}$ where $p = \frac{dy}{dx}$ is

a) $y = cx + \sqrt{a^2c^2 + b^2}$

b) $y = cx - \sqrt{a^2c^2 + b^2}$

c) $y = c - x\sqrt{a^2c^2 + b^2}$

d) $y = c + \sqrt{a^2c^2 + b^2}$.

iv) The differential equation $f(x, y)\frac{dy}{dx} + g(x, y) = 0$ will be exact if

a) $\frac{\partial f}{\partial y} = \frac{\partial g}{\partial x}$ b) $\frac{\partial f}{\partial x} = \frac{\partial g}{\partial y}$

c) $\frac{\partial^2 f}{\partial x^2} = \frac{\partial^2 g}{\partial y^2}$ d) $\frac{\partial^2 f}{\partial y^2} = \frac{\partial^2 g}{\partial x^2}$.

v) The value of $\Gamma(m)\Gamma(1-m)$ is

a) $\frac{2\pi}{\sin \pi}$ b) $\frac{3\pi}{\sin m\pi}$

c) $\frac{\pi}{\sin m\pi}$ d) $\frac{\pi}{\cos m\pi}$.

vi) The minimum number of pendant vertices in a tree with five vertices is

a) 1 b) 2

c) 3 d) 4.

vii) A closed walk in which no vertex (except its terminal vertices) appear more than once is called a/an

a) Path b) Euler line

c) Hamiltonian circuit d) Circuit.

viii) A binary tree should have at least

a) one vertex b) two vertices

c) three vertices d) four vertices.

ix) The degree of a pendant vertex is

a) 0 b) 1

c) 2 d) none of these.

x) The general solution of the differential equation $y = x\frac{dy}{dx} + \sqrt{3\left(\frac{dy}{dx}\right)^2 + 7}$ is

a) $x - cy + 5 = 0$ b) $y = cx + \sqrt{3c^2 + 7}$

c) $y^2 = cx + \sqrt{3c^2 + 7}$ d) none of these.

xi) Integrating factor of the differential equation $\cos x\frac{dy}{dx} + y\sin x + 1$ is

a) $\tan x$ b) $\cos x$

c) $\sec x$ d) $\sin x$.

xii) The order and degree of the differential equation $\left(\frac{d^7y}{dt^7}\right)^2 + \left(\frac{dy}{dt}\right)^6 = y^{14}$ is

a) (2, 7) b) (14, 7)

c) (7, 2) d) (7, 14).

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Solve the ordinary differential equation $y'' - 3y' + 2y = 4t + e^{3t}$, $y(0) = 0$, $y'(0) = -1$ by Laplace transform method.

3. Prove that a graph G has a spanning tree if and only if G is connected.

4. Solve : $(\sin x\cos y + e^{2x})dx + (\cos x\sin y + \tan y)dy = 0$.

5. Prove that the number of odd vertices in a graph is even.

6. Solve : $(D^2 - 5D + 6)y = e^x\cos x$.

GROUP – C
(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Solve : $(2x+3)^2 \frac{d^2y}{dx^2} - (2x+3)\frac{dy}{dx} - 12y = 6x$.

   b) Apply convolution theorem to find inverse Laplace transform of $\frac{1}{(s-1)^5(s+2)}$ where is the transform parameter.

   c) Using Laplace transforms prove that $\int_0^\infty \frac{\sin t}{t}\,dt = \frac{\pi}{2}$.

   5 + 7 + 3

8. a) Using the method of variation of parameter find the complete solution of $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 9e^x$.

   b) Solve : $\frac{dx}{dt} + 2y = e^t$, $\frac{dy}{dt} - 2x = e^{-t}$.

   c) Solve : $\left(x^2D^2 - 3xD + 4\right)y = (1+x)^2$. 5 + 5 + 5

9. a) Solve : $\frac{dy}{dx} - \frac{\tan y}{1+x} = (1+x)e^x \sec y$.

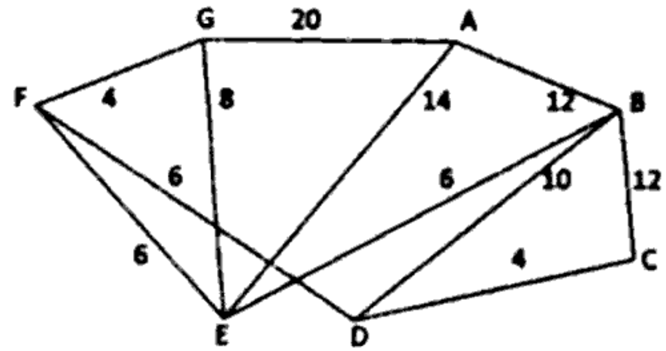
   b) Solve : $y = px + p - p^2$, where $p = \frac{dy}{dx}$.

   c) Solve : $\left(x^4e^x - 2mxy^2\right)dx = 2mx^2y\,dy = 0$. 5 + 5 + 5

10. a) Evaluate $\int_0^3 \frac{dx}{1+\cos x}$, if possible.

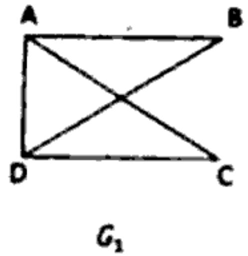
    b) Evaluate $\int_0^{\frac{\pi}{2}} \sqrt{\tan x}\,dx$ by using beta-gama function.

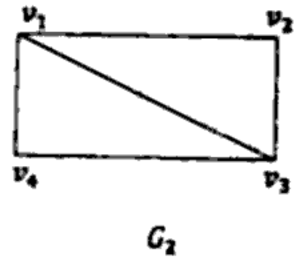
    c) Find by Prim's algorithm a minimal spanning tree from the following graph :



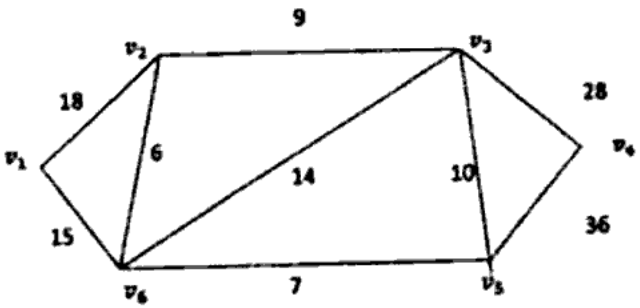
5 + 5 + 5

11. a) Show that the following two graphs are isomorphic :





b) Applying Dijkstra's Algorithm find the shortest path from the vertex v_1 to v_4 in the following simple graph :



c) Draw the graph whose incidence matrix is given below :

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & -1 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 \end{pmatrix}$$

4 + 5 + 6