Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS / B.TECH (CSE/IT) / SEM-4 / M-401/ 2011

2011

MATHEMATICS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following :

$10 \times 1 = 10$

i) A group contains 12 elements. Then the possible number of elements in a subgroup is

a) 3 b) 5

c) 7 d) 11.

ii) In a group (G, 0) if $(a \circ b)^{-1} = a^{-1} \circ b^{-1}$, then

a) G is finite b) G is infinite

c) G is abelian d) none of these.

iii) The mapping $f : \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = |x|,\ x \in \mathbb{R}$ is

a) injective b) surjective

c) bijective d) none of these.

iv) The relation $\{ (a, b) : a, b \in z, ab > 0 \}$ defined on z (the set of integers) is

a) symmetric b) reflexive

c) anti-symmetric d) equivalence.

v) The number of unit elements of the ring (z, +, $\cdot$) is

a) 2 b) 3

c) 1 d) infinite.

vi) If $F : G \rightarrow G'$ be a homomorphism and e is positive identity element of G then $f(e)$ is

a) identity element of G

b) identity element of G'

c) inverse of each element of G'

d) none of these.

vii) Number of operations required in a Boolean Algebra is

a) 1 b) 2

c) 3 d) 4.

viii) The Boolean function $(x'y' + xy + x'y)$ is equivalent to

a) $x' + y'$ b) $x + y$

c) $x' + y$ d) none of these.

ix) The generating function of $(1, 1, \frac{1}{2!}, \frac{1}{3!}, \frac{1}{4!}, \frac{1}{5!}, \ldots \infty)$ is

a) $-\log_e (1 - x)$ b) $\log_e (1 + x)$

c) e^x d) none of these.

x) The solution of the recurrence relation $S_n = 2S_{n-1}$ with $S_0 - 1$ is $S_n =$

a) 2^n b) 2^{n-1}

c) 2^{n+1} d) none of these.

xi) The maximum number of edges in a simple connected graph with n vertices is

a) $2 \bullet {}^nC_2$ b) ${}^nC_2$

c) $(n - 1)$ d) none of these.

xii) A complete graph is

a) regular b) connected

c) simple d) circuit.

GROUP – B

(Short Answer Type Questions)

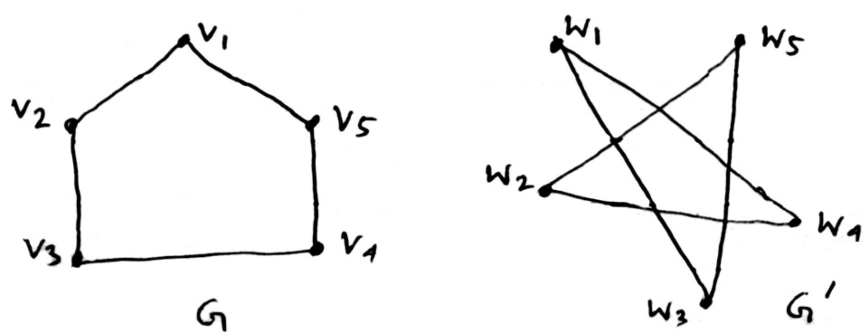
Answer any *three* of the following. $3 \times 5 = 15$

2. If $f : G \to G'$ be a group homomorphism from a group G to the Group G', then show that $ker f$ is a normal subgroup of G.

3. If in a ring R with unity, $(xy)^2 = x^2 y^2$, for all $x, y \in R$ then show that R is commutative.

4. Using generating function, find the integral solutions of $x_1 + x_2 + x_3 + x_4 + x_5 = 10$, whenever, $1 \le x_i \le 5$; $i = 1, 2, \ldots, 5$.

5. Define isomorphism of graph. Show that the graphs G and G' are isomorphic.



6. Show that the number of pendent vertices in a binary tree is $(n + 1) / 2$, where n is the number of vertices in the tree.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Prove that the relation ρ defined on z by $a \rho b$ iff $a^2 \equiv b^2 (\bmod 5), a, b \in z$ is an equivalence relation and also find all equivalence classes.

   b) Define normal subgroup of a group. If G is a group and H is a subgroup of index 2 in G, prove that H is a normal subgroup of G.

   c) Let G be a group. If $a,\ b \in G$ such that $a^4 = e$, the identity element of G and $ab = ba^2$, prove that $a = e$.

   5 + 5 + 5

8. a) If two operations * and 0 on the set Z of integers are defined as follows : $a * b = a + b - 1$, $a \, o \, b = a + b - ab$, prove that $(Z, *, o)$ is commutative ring with unit element.

   b) Construct a simple logic circuit for each of the Boolean functions :

      i) $xy' + x'yz + x'y'z$

      ii) $(yx + xz)z'$.

   c) Using generating function, solve the recurrence relation $a_n - 7a_{n-1} + 10a_{n-2} = 0$ for $a > 1$ and $a_0 = 3,\ a_1 = 3$.

   5 + 5 + 5

9. a) Prove that the intersection of two subrings is a subring.

b) Find the disjunctive normal form (sum of product) for the Boolean expression $(x + y + z) \cdot (xy + x'z)'$.

c) Prove that every cut set in a connected graph contains at least one branch of every spanning tree of the graph.

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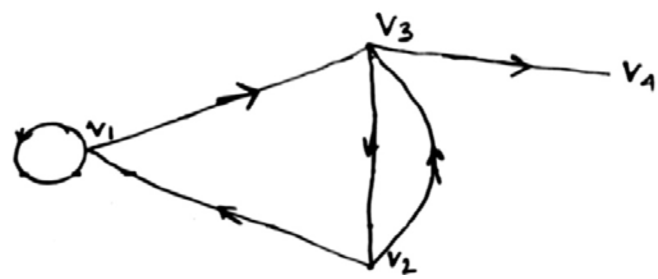
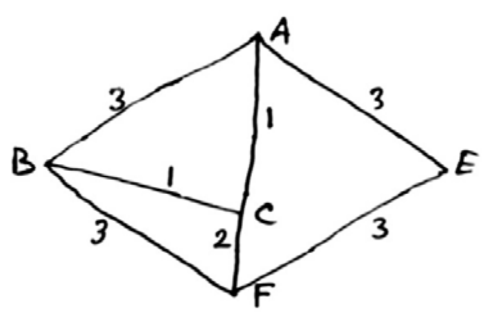
10. a) Construct the Adjacency matrix of the following di-graph :



b) Prove that a tree with n number of vertices has $(n - 1)$ number of edges.

c) Find by Kruskal's Algorithm a minimal spanning tree for the following graph :



5 + 5 + 5

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