



Name : .....

Roll No. : .....

Invigilator's Signature : .....

**CS/B.Tech(CSE/IT)/SEM-3/M-301/2009-10**

**2009**

**MATHEMATICS**

Time Allotted : 3 Hours

Full Marks : 70

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

**GROUP – A**

**( Multiple Choice Type Questions )**

1. Choose the correct alternatives for any *ten* of the following :

$$10 \times 1 = 10$$

- i) If  $A$  and  $B$  are any two events such that  $P(A \cap B) = \frac{1}{2}$ ,  $P(A^C \cap B^C) = \frac{1}{3}$  and  $P(A) = P(B) = p$ , then the value of  $p$  is

- a)  $\frac{7}{12}$                                       b)  $\frac{5}{6}$   
c)  $\frac{1}{3}$                                         d)  $\frac{1}{2}$ .

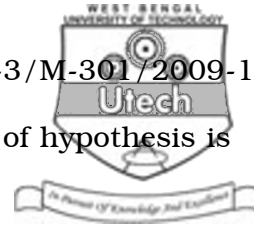
- ii) If the exponential distribution is given by the probability density function

$$f(x) = e^{-x}, 0 < x < \infty,$$

then the mean of the distribution is

- a) 1    b) 3  
c)  $\frac{1}{3}$     d) 4.

- 2



- ix) The power of a test in case of testing of hypothesis is
- $1 - P ( \text{Type I Error} )$
  - $1 - P ( \text{Type II Error} )$
  - $1 - P ( \text{Type I Error} ) P ( \text{Type II Error} )$
  - $P ( \text{Type I Error} ) P ( \text{Type II Error} ) .$
- x) The standard deviation of a sample mean for SRSWR is
- $\sigma^2 / n$
  - $\sigma / \sqrt{n}$
  - $\sigma / n$
  - $n.$
- xi) The maximum likelihood estimate is a solution of the equation
- $\frac{\partial L( \theta )}{\partial \theta} = 0$
  - $\frac{\partial L( \theta )}{\partial \theta} = \text{constant}$
  - $\frac{\partial L( \theta )}{\partial \theta} = \theta$
  - none of these.
- xii) A statistic  $t$  is said to be an unbiased estimator of a population parameter  $\theta$  when
- $E ( t ) = \theta$
  - $E ( t^2 ) = \theta$
  - $E ( t^2 ) = [ E ( \theta ) ]^2$
  - $[ E ( t ) ]^2 = E ( t^2 ) .$
- xiii) The probability  $P ( a < x \leq b )$  ( where  $F ( x )$  is the distribution function of the random variable  $x$  ) is given by
- $F ( b ) - F ( a )$
  - $F ( b ) + F ( a )$
  - $F ( a ) - F ( b )$
  - $F ( a ) F ( b ) .$



**GROUP – B**

**( Short Answer Type Questions )**

Answer any *three* of the following.

$$3 \times 5 = 15$$

2. If  $A$  and  $B$  are two events such that  $P ( A^C \cup B^C ) = 5/6$ ,  $P ( A ) = 1/2$  and  $P ( B^C ) = 2/3$ , show that  $A$  and  $B$  are independent.
3. A random variable  $X$  has the following probability function :  

$X =$	$-2$	$-1$	$0$	$1$	$2$	$3$
$P (X) =$	$0.1$	$k$	$0.2$	$2k$	$0.3$	$3k$

  - i) Calculate  $k$
  - ii) Find  $P ( X < 2 )$ ,  $P ( X \geq 2 )$ ,  $P ( -2 < X \leq 2 )$ .
4. If the chance of being killed by flood during a year is  $1/3000$ , use Poisson distribution to calculate probability that out of 3000 persons living in a village, at least one will die in flood in a year.
5. A random sample with observations 65, 71, 64, 71, 70, 69, 64, 63, 67, 68 is drawn from a normal population with standard deviation  $\sqrt{7.056}$ . Test the hypothesis that the population mean is 69 at 1% level of significance.  
 [ Given :  $P ( 0 < Z < 2.58 ) = 0.495$  ].
6. If  $x$  follows a Normal Distribution with mean 12 and variance 16, find  $P ( x \geq 20 )$ .  
 [ Given :  $\int_{-\infty}^2 \frac{1}{\sqrt{2\pi}} e^{-1/2 t^2} dt = 0.977725$  ]
7. Find the maximum likelihood estimate for the parameter  $\lambda$  of a Poisson distribution on the basis of a sample of size  $n$ .



**GROUP – C**

**( Long Answer Type Questions )**

Answer any *three* of the following.  $3 \times 15 = 45$

8. a) A random variable follows binomial distribution with mean 4 and standard deviation  $\sqrt{2}$ . Find the probability of assuming non-zero value of the variable. 7
- b) Find the mathematical expectation of the number of the points obtained in a single throw of an unbiased die. 8
9. a) A bag contains 7 red and 5 white balls. 4 balls are drawn at random. What is the probability that (i) all of them are red (ii) two of them be white and two red ? 7
- b) If a random variable follows Poisson Distribution such that  $P(1) = P(2)$ ,  
find (i) mean of the distribution  
(ii)  $P(4)$ . 8
10. a) State "Central Limit Theorem".  
A random variable  $x$  has the function  $e^{-x}, x \geq 0$ .  
Show that Tchebycheff's inequality gives  
 $P[|X-1| > 2] < \frac{1}{4}$  and show that actual probability is  
 $e^{-3}$ . 8
- b) If  $T$  is an unbiased estimator of  $\theta$ , show that  $\sqrt{T}$  is biased estimator of  $\theta$ . 7



11. a) A pair of dice is thrown. Find the probability of getting a sum of 7, when it is known that the digit in the first die is greater than that of the second. 5
- b) The manufacturing process of an article consists of two parts  $x$  and  $y$ . The probabilities of defect in parts  $x$  and  $y$  are 10% and 15% respectively. What is the probability that the assembled product will not have any defect ? 5
- c) The probabilities of solving a problem by three students  $A$ ,  $B$  and  $C$  are  $\frac{2}{7}$ ,  $\frac{3}{8}$  and  $\frac{1}{2}$  respectively. If all of them try independently, find the probability that the problem is solved. 5
12. a) The probability density function of a continuous distribution is given by  $f(x) = \frac{3}{4} x(2 - x)$ . Compute mean and variance of the distribution. 5
- b) The mean weight of 500 male students at a certain college is 150 lbs and the standard deviation is 15 lbs. Assuming that the weight is normally distributed find how many students weigh
- between 120 and 155 lbs
  - more than 155 lbs.
- [ Given  $\phi(2) = 0.9772$ ;  $\phi(0.33) = 0.6293$  ] 10



13. a) A survey of 320 families with 5 children each revealed the following distribution :

<i>No. of boys</i>	5	4	3	2	1	0
<i>No. of girls</i>	0	1	2	3	4	5
<i>No. of families</i>	14	56	110	88	40	12

Is the result consistent with the hypothesis that male and female births are equally probable ?

[ Given :  $X^2_{Tab,5\%} = 11.07$  at 5 degrees of freedom ] 8

- b) Intelligence tests on two groups of boys and girls gave the following results :

	Mean	SD	N
Boys	70	20	250
Girls	75	15	150

Is there any significant difference in the mean scores obtained by boys and girls ? 7

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