Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS / B.Tech-(ICE-NEW) / SEM-6 / IC-602 / 2013

2013

ADVANCED CONTROL SYSTEM

Time Allotted : 3 Hours   *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

   i) A matrix is said to be positive semidefinite for all X, if

      a) $\underline{X}^T Q \underline{X} > 0$
      b) $\underline{X}^T Q \underline{X} \geq 0$
      c) $\underline{X}^T Q \underline{X} \neq 0$
      d) $\underline{X} Q \underline{X}^T \geq 0$.

   ii) The sufficient conditions for a relative minimum of a twice differentiable function $f(x)$ at an interior point x^* is

      a) $\frac{\partial f}{\partial x}\Big|_{x^*} > 0$
      b) $\frac{\partial f}{\partial x}\Big|_{x^*} = 0$
      c) $\frac{\partial^2 f}{\partial x^2}\Big|_{x^*} \leq 0$
      d) $\frac{\partial^2 f}{\partial x^2}\Big|_{x^*} > 0$.

iii) In the minimum time problem, if the final state is fired, then the target set is a

a) straight line b) ellipse

c) parabola d) hyperbola.

iv) The variation, δv of the given functional

$$v = \int_0^1 \left(2x^2 + x \right) dt \text{ is}$$

a) $\int_0^1 (4x + 1)\, \delta x \, dt$

b) $\int_0^1 2(x + 1)\, \delta x \, dt$

c) $\int_0^1 4x \, \delta x \, dt$

d) zero.

v) If x^* is a constrained extremum for a Lagrangian $\lambda(x^*, \lambda^*)$, then the condition for optimality is

a) $\frac{\partial L}{\partial x} \Big|_{x^*, \lambda^*} = 0$ b) $\frac{\partial L}{\partial \lambda} \Big|_{x^*, \lambda^*} = 0$

c) both (a) and (b) d) $\frac{\partial^2 L}{\partial x^2} \Big|_{x^*, \lambda^*} = 0$.

vi) The performance index (PI) for the minimization of the total expended energy is

a) $PI = \int_{t_o}^{t_f} \sum_{i=1}^{m} |(u_i^2 t)|^2 \, dt$

b) $PI = \int_{t_o}^{t_f} \sum_{i=1}^{m} u_i^2 t \, dt$

c) $PI = \int_{t_o}^{t_f} \sum_{i=1}^{m} u_i^2 \, dt$

d) $PI = \int_{t_o}^{t_f} \sum_{i=1}^{m} u_i t^2 \, dt$.

vii) Which of the following terms is for sensitivity derivative of the system in the MIT rule described as $\frac{d\theta}{dt} = -\gamma e \frac{\delta e}{\delta\theta}$?

a) γ

b) e

c) $\frac{\delta e}{\delta\theta}$

d) $\frac{d\theta}{dt}$.

viii) The basic objective of the adaptive control system is monitoring of the .... of the control system for .... parametres.

a) performance index, unknown and varying

b) controlled variable, unknown and varying

c) controlled variable, constant and known

d) performance index, constant and unknown.

ix) In sliding phase, which type of stability is measured ?

a) Globally finite stability

b) Relative stability

c) Asymptotic stability

d) Conditional stability.

x) The minimisation of $PI = \int_{t_o}^{t_f} | u_t | \, dt$ for a control input $u(t)$ represents

a) minimum fuel problem

b) minimum time problem

c) minimum energy problem

d) all of these.

xi) In sliding mode control, if $\sigma(x) > 0$, for the function sign $(\sigma(x))$, then the control gain (ρ) would be

a) $\rho > 0$ b) $\rho < 0$

c) $\rho \geq 0$ d) $\rho \leq 0$.

xii) In input/output linearization technique, the number of internal dynamies is

a) $n - r$ b) $n + r$

c) n d) r.

xiii) Minimum degree pole placement algorithm is applicable for

a) Gain scheduling b) Self tuning regulator

c) Dual control d) none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. If a scalar valued function, $f(x) = f(x_1, x_2 \ldots\ldots\ldots\ldots x_n)$ is continuous for all $x \in R^n$, find out the condition for minimization of the function $f(x)$ applying Taylor series expansion method.

3. Find the equation of the curve that is an extremal for the functional $J = \int_0^{t_f} \left(\frac{1}{2} \dot{x}^2 - t\dot{x} + 2x^2 \right) dt$

subject to the boundary conditions $x(0) = 1$, $t_f = 2$, and $x(2) = 10$.

4. Let a function $f(x) = -x_1 x_2$ and $g(x) = x_1^2 + x_2^2 - 1$. Applying Lagrange multiplier method show that the points $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right)$ and $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \right)$ provide the minima of $f(x)$ subject to the constraint given by $g(x) = 0$.

5. Draw the block diagram of self tuning regulator. Describe the MIT rule. 3 + 2

6. What is internal dynamics in input/output linearization method ? Find out the internal dynamic equation for the given system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} \begin{bmatrix} x_2^3 + u \\ u \end{bmatrix} ; \; y = x_1$$

where variables x, u and y have their usual significances.

2 + 3

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. 3 × 15 = 45

7. a) Describe the Euler-Lagrange equation for both end points fixed problems.

b) Optimize the following functional

$$L = \int_0^3 \left(\dot{x}^2(t) + 2x(t)\,\dot{x}(t) + 2x^2(t) \right) dt$$

such that $x(0) = 1$ and $x(3)$ is free. 7 + 8

8. a) Derive the Pontryagin's optimal principle to solve optimal control problems.

b) The performance index of the system is given by

$$J = \int_0^2 \left[x_1^2(t) + u^2(t) \right] dt .$$

The system is decribed by the equation, $\dot{x}_1 = u - x_1$ with initial conditions $x_1(0) = 1$ and $x_1(2) = 0$. Find $u(t)$ to minimize the system considering the above two-point boundary value problem applying Pontryagin's optimal principle. 5 + 10

9. a) Design the control law by input/output linearization method for the system

$$\dot{x}_1 = \sin x_2 + \left(x_2 + 1 \right) x_3$$

$$\dot{x}_2 = x_1^5 + x_3$$

$$\dot{x}_3 = x_1^2 + u$$

$$y = x_1 .$$

b) What is reaching phase in sliding mode control ? Find the general value of reaching time from a Lyapunov function $V(x)$ and the switching variable $\sigma(x)$.

9 + (2 + 4)

10. a) Why feedback linearization is important over Jacobian linearization ?

b) Consider an non-linear system with state equations

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = C_1 \left(1 - x_1^2 \right) x_2 - C_2 x_1 + u$$

and the output equation is $y = x_1$, where $C_1 = 0 \cdot 5$ and $C_2 = 1$ are constants.

i) Represent this non-linear system into "affine form".

ii) Obtain a state feedback control law employing feedback linearization method. 1 + (2 + 12)

11. Write short notes on any *three* of the following : 3 × 5

a) Lagrange, Mayer and Bolza problems

b) State regulator (LQR) problem

c) Principle of optimality

d) Gain scheduling

e) Switching surface.