

Time Allotted : 3 Hours
Full Marks : 70

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

## GROUP - A

## ( Multiple Choice Type Guestions)

1. Choose the correct alternatives for any ten of the following :

$$
10 \times 1=10
$$

i) The probability that a leap-year selected at random will contain 53 sundays is
a) $\frac{3}{7}$
b) $\frac{2}{7}$
c) $\frac{5}{7}$
d) $\frac{4}{9}$.
ii) If a coin is tossed 6 times in succession, the probability of getting at least one head is
a) $\frac{63}{64}$
b) $\frac{3}{64}$
c) $\frac{7}{63}$
d) None of these.

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iii) The probability that the 4 children of a family have different birthdays is
a) 0.9836
b) 0.4735
c) 0.9
d) $0 \cdot 757$.
iv) A tree has $n$ vertices. The number of its edges is
a) $n+1$
b) $n-1$
c) $2 n$
d) none of these.
v) The value of $m$ such that $3 y-5 x^{2}+m y^{2}$ is a harmonic function is
a) 5
b) -5
c) 0
d) 3 .
vi) Let $X$ and $Y$ be two random variables such that $Y=a+b x$ where $a$ and $b$ are constants. Then, $\operatorname{Var}(y)$ is
a) $\quad b^{2} \operatorname{Var}(X)$
b) $\quad \operatorname{Var}(X)$
c) $\quad a^{2} \operatorname{Var}(X)$
d) $(b / a) \operatorname{Var}(X)$.
vii) The value of $\int \frac{\mathrm{d} z}{z+3}$ where $C$ is a circle $|z|=1$ is C
a) 0
b) 1
c) 2
d) -1 .
viii) If $f(z)=\frac{1}{z^{4}-2 z^{3}}$, then $z=0$ is a pole of order
a) 3
b) 2
c) 1
d) 4 .
ix) The order of the pole $z=0$ of the function

a) 1
b) 2
c) 3
d) 4 .
x) The period of the function $f(x)=\sin 2 \pi x$ is
a) $\frac{1}{2}$
b) 1
c) 0
d) $\frac{1}{3}$.
xi) If $f(x)=x \sin x,-\pi \leq x \leq \pi$, be presented in Fourier series as $\frac{a_{o}}{2}+\sum\left(a_{n} \cos n x+b_{n} \sin n x\right)$, $n=1$
then the value of $a_{o}$ will be
a) 2
b) 0
c) 4
d) 1 .
xii) If two variables $x$ and $y$ are uncorrelated, then $r_{x y}$ is
a) 1
b) 2
c) 3
d) 0 .
xiii) If $x=4 y+5$ be a regression line of $x$ on $y$ then by is
a) $\frac{1}{4}$
b) 4
c) 0
d) 1 .

2. Show that $f(x)$ given by

$$
\begin{aligned}
f(x) & =x ; 0<x<1 \\
& =k-x ; 1<x<2 \\
& =0 ; \text { elsewhere },
\end{aligned}
$$

is a probability density function for a suitable value of $k$. Calculate the probability that the random variable lies between $\frac{1}{2}$ and $\frac{3}{2}$.
3. Find the Fourier sine transform of $\frac{e^{-a x}}{x}$.
4. Evaluate $\int \frac{3 z^{2}-2}{z-1} \mathrm{~d} z$, where $c$ is the circle $|z|=\frac{1}{2}$. C
5. An urn contains 3 white and 5 black balls. One ball is drawn and its colour is unnoted, kept aside and then another ball is drawn. What is the probability that it is (i) black (ii) white ?
6. Find the mean and standard deviation of a bionomial distribution.

7. a) If $A$ and $B$ are mutually independent events, prove that $A^{C}$ and $B^{C}$ are also mutually independent events.
b) There are three identical urns containing white and black balls. The first urn contains 3 white and 4 black balls, the 2 nd urn contains 4 white and 5 black balls and the 3 rd urn contains 2 white and 3 black balls. An urn is chosen at random and a ball is drawn from it. If the drawn ball is white, what is the probability that the 2 nd urn chosen ?
c) A random variable $X$ has the following p.d.f.

$$
\begin{aligned}
f(x) & =c x^{2} \quad 0 \leq x \leq 1 \\
& =0, \text { otherwise } .
\end{aligned}
$$

Find (i) $c$ (ii) $P\left(0 \leq X \leq \frac{1}{2}\right)$. $5+5+5$
8. a) Find the Fourier series expansion of the periodic function of period $2 \pi$,
$f(x)=x^{2},-\pi \leq x \leq \pi$. Hence deduce
$\frac{1}{1^{2}}-\frac{1}{2^{2}}+\frac{1}{3^{2}}-\frac{1}{4^{2}}+\ldots \ldots \ldots+\ldots \ldots=\frac{\pi^{2}}{12}$.
b) The following marks have been obtained by students in Mathematics and Statistics (out of 100) :

| Maths | 45 | 55 | 56 | 58 | 60 | 65 | 68 | 70 | 75 | 80 | 85 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Stats | 56 | 50 | 48 | 60 | 62 | 64 | 65 | 70 | 74 | 82 | 90 |

Compute the co-efficient of correlation for the above data. Find also the equations of the lines of regrssion.
9. a) Solve
$\frac{\partial u}{\partial t}=k \frac{\partial^{2} u}{\partial x^{2}}, x>0, t>0$,
if $u(0, t)=0, u(x, 0)=e^{-x}, x>0, u(x, t)$ is unbounded.
b) If $f(z)$ is a regular function of $z$, then prove that $\left(\frac{\partial^{2}}{\partial x^{2}}+\frac{\partial^{2}}{\partial y^{2}}\right) \quad|f(z)|^{2}=4\left|f^{\prime}(z)\right|^{2} . \quad 8+7$
10. a) Apply Dijkstra's algorithm to determine a shorterst path between $s$ to $z$ in the following graph :

## Dia.

b) Define isomorphism of two graphs. Examine whether the following graph $G$ and $G^{\prime}$ are isomorphic. Give reasons.

## Dia.

$$
8+7
$$

11. a) Use residue theorem to evaluate $\int \frac{3 z^{2}+z-1}{\left(z^{2}+1\right)(z-3)} \mathrm{d} z$
C
around the circle $|z|=2$.
b) Expand the function $f(z)=\frac{1}{\left(z^{2}+1\right)(z+2)}$ in the region $|z|<1$.
c) Show that the function $f(z)= \begin{cases}\frac{3 x y^{2}}{x^{2}+y^{2}} & \text { for } z \neq 0 \\ 0 & \text { for } z=0\end{cases}$ is continuous at $z=0$. $5+7+3$
12. a) Show that a simple graph with $n$ vertices and $k$-components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges.
b) Find the incidence matrix of the following graph.

## Dia.

c) Find the Fourier sine transform of the functon

$$
f(x)= \begin{cases}1 & \text { for } 0<x \leq \pi \\ 0 & \text { for } x>\pi\end{cases}
$$

and hence evaluate the integral

$$
\int^{\infty} \frac{1-\cos p \pi}{p} \sin p x d p
$$

$$
5+5+5
$$

0

