



Name :

Roll No. :

Invigilator's Signature :

**CS/B.Tech(ECE)/SEM-7/EC-704B/2011-12
2011**

**ADVANCED ENGINEERING MATHEMATICS FOR
ELECTRONICS ENGINEERING**

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following :

$$10 \times 1 = 10$$

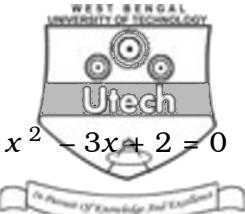
i) Cauchy-Riemann equations are

a) $u_x = -v_y$, $u_y = v_x$

b) $u_x = v_y$, $u_y = -v_x$

c) $u_x = v_y$, $u_y = v_x$

d) $u_x = -v_y$, $u_y = -v_x$.



ii) If α, β are the roots of the equation $x^2 - 3x + 2 = 0$

$$\begin{array}{c|ccc} & 0 & \alpha & \beta \\ \text{then } & \beta & 0 & 0 \\ & 1 & -\alpha & \alpha \end{array} \quad \text{is}$$

iii) The residue of $f(z) = \frac{1}{(z+1)^2(z-2)}$ at $z = -1$ is

- a) $\frac{1}{3}$ b) $\frac{1}{9}$
 c) $-\frac{1}{3}$ d) $-\frac{1}{9}$.

iv) If $L [f(t)] = F(s)$ then $L [f(2t)]$ will be

- a) $\frac{1}{2} F\left(\frac{s}{2}\right)$ b) $2F\left(\frac{s}{2}\right)$
 c) $\frac{1}{2} F(2s)$ d) $2 F(2s)$.

v) $L \{ e^{-2t} \cos t \}$ is equal to

- a) $\frac{p}{p^2 + 4p + 5}$

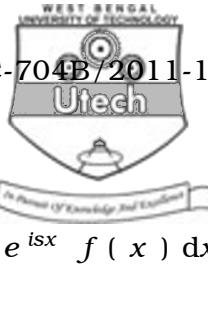
b) $\frac{p+2}{p^2 + 4p + 5}$

c) $\frac{p+3}{p^2 + 4p + 5}$

d) $\frac{p+1}{p^2 + 4p + 5}$.

vi) The differential equation $p^2 + q^3 - pq + zxy = 0$ is

- a) degree = 1, order = 1, linear
 - b) degree = 2, order = 2, linear
 - c) degree = 3, order = 1, non-linear
 - d) degree = 4, order = 2, non-linear.



vii) If the Fourier transform of a function

$$f(x) \text{ is } \bar{F}(s) = F\{f(x)\} = \int_{-\infty}^{\infty} e^{isx} f(x) dx,$$

then the inversion formula for Fourier transform is

$$\text{a) } \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-isx} \bar{F}(s) ds$$

$$\text{b) } \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-isx} \bar{F}(s) ds$$

$$\text{c) } \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{isx} \bar{F}(s) ds$$

$$\text{d) } \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{isx} \bar{F}(s) ds.$$

viii) For Legendre polynomial of degree n , $P_2(x)$ is equal to

$$\text{a) } 1 \qquad \qquad \qquad \text{b) } \frac{1}{2}(3x^2 - 1)$$

$$\text{c) } \frac{1}{2}(x^2 - 3) \qquad \qquad \text{d) } \frac{1}{2}(5x^3 - 3x).$$

ix) The equation $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + (x^2 - 1)y = 0$ is

a) Bessel's equation of order zero

b) Bessel's equation of order one

c) Legendre's equation of order zero

d) Legendre's equation of order one.



x) The solution of the differential equation

$$\left(\frac{y^2 z}{x} \right) p + xzq = y^2 \text{ is}$$

- a) $\phi(x^2 - z^2, x^3 - y^3) = 0$
- b) $\phi(x^2 - z^2, x^2 - y^2) = 0$
- c) $\phi(x^2 - z^2, x - y) = 0$
- d) $\phi(x^2 - z^2, 1) = 0.$

xi) Let $f(z) = \sin \frac{1}{z}$. Then $Z = 0$ is

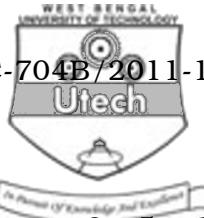
- a) a pole
- b) removable singularity
- c) essential singularity
- d) none of these.

xii) The bilinear transformation that maps the points

$$z_1 = 0, z_2 = -i, z_3 = -1 \text{ into } w_1 = i, w_2 = 1, w_3 = 0$$

respectively is

- a) $w = \left(\frac{z+1}{z-1} \right)$
- b) $w = i \left(\frac{z-1}{z+1} \right)$
- c) $w = -i \left(\frac{z-1}{z+1} \right)$
- d) $w = -i \left(\frac{z+1}{z-1} \right).$

**GROUP - B****(Short Answer Type Questions)**Answer any *three* of the following.

$$3 \times 5 = 15$$

2. Show that $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cdot \cos x$.

3. Solve $4 \frac{\partial^2 z}{\partial x^2} - 4 \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 16 \log(x + 2y)$.

4. Determine the row rank and the column rank of the matrix A and verify that the row rank of the matrix A equals to column rank of the matrix A , where

$$A = \begin{pmatrix} 2 & 1 & 4 & 3 \\ 3 & 2 & 6 & 9 \\ 1 & 1 & 2 & 6 \end{pmatrix}$$

5. If $f(z)$ is analytic, prove that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) |f(z)|^2 = 4 |f'(z)|^2.$$

6. Evaluate $L^{-1} \left\{ \frac{1}{s^3 (s+1)^3} \right\}$ using convolution theorem.

 $3 \times 15 = 45$ **GROUP - C****(Long Answer Type Questions)**Answer any *three* of the following.

7. a) Prove that if A & B are orthogonal matrices of the same order, then AB is also orthogonal. 3

- b) Find the rank of the matrix

$$\begin{pmatrix} 0 & 0 & 2 & 2 & 0 \\ 1 & 3 & 2 & 4 & 1 \\ 2 & 6 & 2 & 6 & 2 \\ 3 & 9 & 1 & 10 & 6 \end{pmatrix}$$

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- c) Find the Fourier sine integral for $f(x) = e^{-\beta x}$, hence

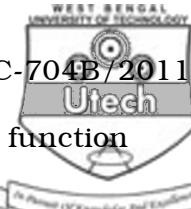
$$\text{show that } \frac{\pi}{2} \cdot e^{-\beta x} = \int_0^{\infty} \frac{\lambda \sin \lambda x}{\beta^2 + \lambda^2} d\lambda. \quad 6$$

8. a) Show that the line $y = \frac{x}{3}$ is mapped onto the circle under the bilinear transformation $w = \frac{iz+2}{4z+i}$. Find the centre and radius of the image circle. 5

- b) Applying residue theorem evaluate

$$\int_C \frac{3z^2 + z - 1}{(z^2 - 1)(z - 3)} dz \text{ where } c \text{ is the circle } |z| = 2. \quad 5$$

- c) Prove that $\int_0^\alpha \frac{\cos x}{1+x^2} dx = \frac{\pi}{2e}$. 5



9. a) Find the Fourier cosine transform of the function

$$f(x) = \frac{1}{1+x^2}.$$

5

- b) Applying binomial theorem $(x^2 - 1)^n$, differentiating n times term by term and comparing with Legendre coefficient, prove the Rodrigues Formula

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} [(x^2 - 1)^n]. \quad 5$$

- c) Find the Laurent's series expansion of $f(z) = z^2 e^{\frac{1}{z}}$. 5

10. a) Show that generating function for Bessel Function

$$J_n(x) \text{ is } e^{x/2(t - \frac{1}{t})}. \quad 10$$

- b) Prove that $J_{3/2} = \sqrt{\frac{2}{\pi x}} \left(\frac{\sin x}{x} - \cos x \right)$. 5

11. a) Solve by Fourier transform

$$K \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \quad 0 < x < \infty, \quad t > 0$$

with $u(0, t) = 0, t > 0$

$$u(x, 0) = f(x), \quad 0 < x < \infty$$

and $u \& \frac{\partial u}{\partial x} \not\equiv 0$ as $x \not\equiv \infty$. 9

- b) Using contour integration evaluate

$$\int_0^\pi \frac{1 + 2 \cos \theta}{5 + 4 \cos \theta} d\theta. \quad 6$$

