



Name :

Roll No. :

Invigilator's Signature :

CS / B.TECH (CT) / SEM-3 / M (CT) 301/ 2010-11

2010-11

APPLIED MATHEMATICS

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

GROUP – A

(Multiple Choice Type Questions)

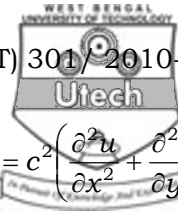
1. Choose the correct alternatives for any *ten* of the following :

$$10 \times 1 = 10$$

i) The point where the function $f(x) = |z|^2$ is not analytic
is

- a) Entire complex plane
- b) Entire complex plane except at $z = 0$
- c) $z = 0$
- d) No such point exist.

- 2



vi) A two dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$

becomes Laplace equation when

- a) 'u' does not depend on time
 - b) 'u' does not depend on space
 - c) $c = 0$
 - d) $c = 1$.
- vii) $\sin 3x$ is a periodic function of period

- a) $\frac{\pi}{3}$
- b) 2π
- c) 6π
- d) $\frac{2\pi}{3}$.

viii) If the function $f(x)$ satisfies Dirichlet's condition and $x = c$ is a point of discontinuity, then the Fourier series of $f(x)$ at the point $x = c$ converges to

- a) $f(c)$
- b) $\frac{1}{2}[f(c-0) + f(c+0)]$
- c) $\frac{1}{2}[f(-c+0) + f(c-0)]$
- d) $\frac{1}{2}[f(c+0) + f(-c-0)]$.



ix) $f(x) = 1 \quad 0 \leq x < 3$
 $= -1 \quad -3 < x < 0$

is an example of the

- a) odd function
 - b) even function
 - c) neither odd nor even function
 - d) periodic function.
- x) If A and B be two events then which one of the following is false ?
- a) $P(A + B) + P(AB) \leq 1$ b) $P(A + B) \leq P(A) + P(B)$
 - c) $P(AB) \geq P(A) + P(B)$ d) $P(A + B) + P(AB) \geq 0$.
- xi) If the Fourier Transform of $f(x)$, $\mathcal{F}\{f(x)\} = F(s)$ then the Fourier Transform of $\mathcal{F}\{f(x)\cos ax\}$ is
- a) $F(s - a)$ b) $F(s + a)$
 - c) $\frac{1}{2}\{F(s - a) + F(s + a)\}$ d) $F(s - a) + F(s + a)$.
- xii) What is the probability of not getting a double six in a throw with two dice ?
- a) $1/36$ b) $35/36$
 - c) $1/18$ d) $25/36$.



GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following.

$$3 \times 5 = 15$$

2. Verify whether the complex valued function $f(z) = \bar{z}$ is analytic at $z = 0$.
3. Find the Taylor's series expansion of the function $f(z) = \ln z$ about $z = 1$. Find also the region of convergence.
4. Solve the p.d.e. $p^2 + q^2 = 1$.
5. Find the Fourier series of the following function by extending it to a periodic function :

$$f(x) = 3 \quad 0 < x \leq 5$$

$$= -3 \quad -5 < x \leq 0$$

Show that the Fourier series does not converge to $f(x)$ at $x = 0$.

6. Show that the probability of occurrence of only one of the events 'A' and 'B' is

$$P(A) + P(B) - 2P(AB)$$

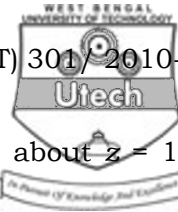
GROUP – C

(Long Answer Type Questions)

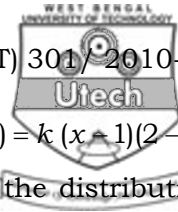
Answer any *three* of the following.

$$3 \times 15 = 45$$

7. a) Show that the function $u = e^{-y} \cos x$ is harmonic and find the corresponding analytic function $f(z)$. What is the imaginary part of this analytic function ?



- b) Expand the function $f(z) = \frac{z}{(z-1)(z-3)}$ about $z = 1$ in the region $0 < |z-1| < 2$. Which kind of series is it ? Find also the residue term. 9 + 6
8. a) Find the integral surface of the p.d.e. $(y-z)p + (z-x)q = x-y$, which passes through the curve $xy = 4, z = 0$.
- b) The ends $x = 0$ and $x = l$ of a finite wire are maintained at zero temperature. Given that the temperature $u(x, t) = f(x)$ at $t = 0$. Determine the temperature at a subsequent time 't'. [Assume c^2 is the diffusivity of the material of the wire]. 7 + 8
9. a) Find a Fourier series of the function $f(x) = x - x^2$, $-\pi < x \leq \pi$.
Hence find the value of the series
$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$$
- b) Find the sine series which represents the function $f(x) = \pi - x$ in $0 < x < \pi$. 10 + 5
10. a) Describe the Newton-Raphson method for solving an algebraic equation.
- b) Using Euler's method with $h = 0.1$, find the solution of $\frac{dy}{dx} = x^2 + y^2$, $y(0) = 0$ at $x = 0.6$.
[Correct up to 3 decimal places] 7 + 8



11. a) The p.d.f. of a random variable X is $f(x) = k(x-1)(2-x)$ for $1 \leq x \leq 2$. Determine the value of k , the distribution function $F(x)$ and $P\left(\frac{5}{4} \leq X \leq \frac{3}{2}\right)$.
- b) If X has a binomial distribution with parameter ' n ' and ' p ', then show that
- its mean is np and
 - variance is npq .

7 + 8

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