Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS / B.TECH (CT) / SEM-3 / M (CT) 301/ 2010-11

2010-11

APPLIED MATHEMATICS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following :

$10 \times 1 = 10$

i) The point where the function $f(x) = |z|^2$ is not analytic is

a) Entire complex plane

b) Entire complex plane except at $z = 0$

c) $z = 0$

d) No such point exist.

ii) Evaluate $\oint_C \frac{\sin z}{z^2} dz$, where $C : |z| = 4$:

a) 0 b) $2\pi i$

c) 1 d) 2π.

iii) A function $f(z)$ can be expanded in a region $|z| > a$ [if $f(z)$ is analytic in the said region] by a

a) Taylor series

b) Laurent's series

c) neither Tailor series nor Laurent's series

d) either Taylor series or Laurent's series.

iv) Evaluate $px - z = e^q - qy$:

a) $z = ax + by + e^b$ b) $z = ax + by + e^a$

c) $z = ax + by - e^b$ d) $z = ax + by - e^a$.

v) The solution of a p.d.e. given by $z = f(p, q, x, y)$ is a

a) straight line b) curve

c) surface d) volume.

vi) A two dimensional heat equation $\frac{\partial u}{\partial t} = c^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$ becomes Laplace equation when

a) 'u' does not depend on time

b) 'u' does not depend on space

c) $c = 0$

d) $c = 1$.

vii) $\sin 3x$ is a periodic function of period

a) $\frac{\pi}{3}$ b) 2π

c) 6π d) $\frac{2\pi}{3}$.

viii) If the function $f(x)$ satisfies Derichlet's condition and $x = c$ is a point of discontinuity, then the Fourier series of $f(x)$ at the point $x = c$ converges to

a) $f(c)$ b) $\frac{1}{2}[f(c-0) + f(c+0)]$

c) $\frac{1}{2}[f(-c+0) + f(c-0)]$ d) $\frac{1}{2}[f(c+0) + f(-c-0)]$.

ix) $f(x) = 1 \qquad 0 \le x < 3$

$= -1 \qquad -3 < x < 0$

is an example of the

a) odd function

b) even function

c) neither odd nor even function

d) periodic function.

x) If A and B be two events then which one of the following is false ?

a) $P(A+B)+P(AB) \le 1$ b) $P(A+B) \le P(A)+P(B)$

c) $P(AB) \ge P(A)+P(B)$ d) $P(A+B)+P(AB) \ge 0$.

xi) If the Fourier Transform of $f(x)$, $\mathcal{F}\{f(x)\} = F(s)$ then the Fourier Transform of $\mathcal{F}\{f(x)\cos ax\}$ is

a) $F(s-a)$ b) $F(s+a)$

c) $\frac{1}{2}\{F(s-a)+F(s+a)\}$ d) $F(s-a)+F(s+a)$.

xii) What is the probability of not getting a double six in a throw with two dice ?

a) 1/36 b) 35/36

c) 1/18 d) 25/36.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Verify whether the complex valued function $f(z) = \bar{z}$ is analytic at $z = 0$.

3. Find the Taylor's series expansion of the function $f(z) = ln\, z$ about $z = 1$. Find also the region of convergence.

4. Solve the p.d.e. $p^2 + q^2 = 1$.

5. Find the Fourier series of the following function by extending it to a periodic function :

   $f(x) = 3 \qquad 0 < x \leq 5$

   $= -3 \qquad -5 < x \leq 0$

   Show that the Fourier series does not converge to $f(x)$ at $x = 0$.

6. Show that the probability of occurrence of only one of the events 'A' and 'B' is

   $P(A) + P(B) - 2P(AB)$

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Show that the function $u = e^{-y} \cos x$ is harmonic and find the corresponding analytic function $f(z)$. What is the imaginary part of this analytic function ?

b) Expand the function $f(z) = \dfrac{z}{(z-1)(z-3)}$ about $z = 1$ in the region $0 < |z-1| < 2$. Which kind of series is it ? Find also the residue term. 9 + 6

8. a) Find the integral surface of the p.d.e. $(y-z)p + (z-x)q = x-y$, which passes through the curve $xy = 4$, $z = 0$.

b) The ends $x = 0$ and $x = l$ of a finite wire are maintained at zero temperature. Given that the temperature $u(x,t) = f(x)$ at $t = 0$. Determine the temperature at a subsequent time 't'. [Assume c^2 is the diffusivity of the material of the wire]. 7 + 8

9. a) Find a Fourier series of the function $f(x) = x - x^2$, $-\pi < x \leq \pi$.

Hence find the value of the series

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots.$$

b) Find the sine series which represents the function $f(x) = \pi - x$ in $0 < x < \pi$. 10 + 5

10. a) Describe the Newton-Raphson method for solving an algebraic equation.

b) Using Euler's method with $h = 0{\cdot}1$, find the solution of

$$\frac{dy}{dx} = x^2 + y^2,\ y(0) = 0 \text{ at } x = 0{\cdot}6.$$

[Correct up to 3 decimal places] 7 + 8

11. a) The p.d.f. of a random variable X is $f(x) = k(x-1)(2-x)$ for $1 \le x \le 2$. Determine the value of k, the distribution function $F(x)$ and $P\left(\frac{5}{4} \le X \le \frac{3}{2}\right)$.

b) If X has a binomial distribution with parameter 'n' and 'p' , then show that

i) its mean is np and

ii) variance is npq. 7 + 8

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