# 2014

## Operations Research

*Time Alloted : 3 Hours* *Full Marks : 70*

***The figure in the margin indicate full marks.***
***Candidates are required to give their answers in their own words as far as practicable***

### GROUP - A
### ( Multiple Choice Type Questions )

**1. Choose the correct alternatives for any ten of the following:** **10x1=10**

i) **The possible number of basic solutions in a system of m equations in n unknowns will be**

a) $\frac{n!}{m!(n-m)!}$ b) $\frac{(n-m)!}{n!m!}$

c) $\frac{n!m!}{(n-m)!}$ d) $\frac{m!}{n!(m-n)!}$

ii) **In a (M/M/1 : $\infty$ /FIFO) queue model with arrival and service rate $\lambda$ and $\mu$ $(\lambda < \mu)$ respectively, then the 'Average length of the non-empty queue' [E(m/m>0] is given by**

a) $\lambda/(\mu-\lambda)$ b) $\mu/(\mu-\lambda)$

c) $1/(\mu-\lambda)$ d) $1/(\lambda-\mu)$

iii) The shortest path between any two nodes in a Network is determined by

a) Dijkastra's algorithm b) Floyd's algorithm
c) Critical path method d) none of these

iv) Which one of the following is not a deterministic method

a) L.P.P. b) T.P.
c) C.P.M. d) P.E.R.T.

v) The basic feasible solution of the system of equations

$x_1 + x_2 + x_3 = 8$

$3x_1 + 2x_2 = 18$ are

a) No basic solution b) (2, 6, 0), (6, 0, 2)
c) (1, 7, 0), (7, 1, 0) d) None of these

vi) In EOQ inventory problem with no shortage, in which demand is assumed to be fixed and completely pre – determined, the economic lot size is

a) $\sqrt{\frac{2DC_3}{C_1}}$ b) $\sqrt{\frac{2DC_1}{C_3}}$

c) $\sqrt{2DC_3C_1}$ d) None of these

vii) In PERT analysis which of the following distribution is assumed for the duration of the activity?

a) Normal b) Beta
c) Poisson d) Exponential.

viii) The value of the game having the following pay-offs matrix is

| | $B_1$ | $B_2$ | $B_3$ |
|---|---|---|---|
| $A_1$ | 10 | 2 | 3 |
| $A_2$ | 7 | 6 | 8 |
| $A_3$ | 0 | 3 | 1 |

a) 6 b) 10
c) 3 d) 2

ix) In a simple deterministic EOQ model, with constant demand rate (D) and infinite rate of production, the economic lot size is

a) $\sqrt{2K/Dh}$ b) $\sqrt{2/KDh}$
c) $\sqrt{2KDh}$ d) $\sqrt{2KD/h}$

x) In an assignment problem, the minimum number of lines covering all the zeros in the reduced cost matrix of order n can be

a) at most n b) n+1
c) n-1 d) at least n

xi) To find critical path, in the forward pass calculations, we calculate for every tail node of each activity the

a) Latest start time b) Latest finish time
c) Earliest start time d) Earliest finish time

xii) The time complexity of the Floyd's algorithm is

a) O(n) b) $O(n^2)$
c) $O(n^3)$ d) $O(n^2 \log n)$

## GROUP - B

### ( Short answer type Questions )

### Answer any three Questions

**3x5=15**

2. **Solve graphically the following LPP**

**Maximize $z = 3x_1 + 2x_2$**

**Sub. to constraints:**

$$-2x_1 + x_2 = 1,$$

$$x_1 + x_2 \leq 3,$$

$$x_1 \leq 2,$$

$$x_1, x_2 \geq 0$$

3. **Customer arrives at a sales counter managed by a single person according to a poisons process with a mean rate of 20 per hour. The time required to serve customer has an exponential distribution with a mean 100 seconds. Find the average waiting time of a customer.** **5**

4. **Find the dual of the following LPP**

**Maximize** $Z = 2x_1 + 7x_2 + 5x_3$

**Subject to** $2x_1 + 5x_2 + 7x_3 \leq 17$

$3x_1 + 2x_2 + 5x_3 = 13$

$5x_1 + 3x_2 + x_3 \leq 9$

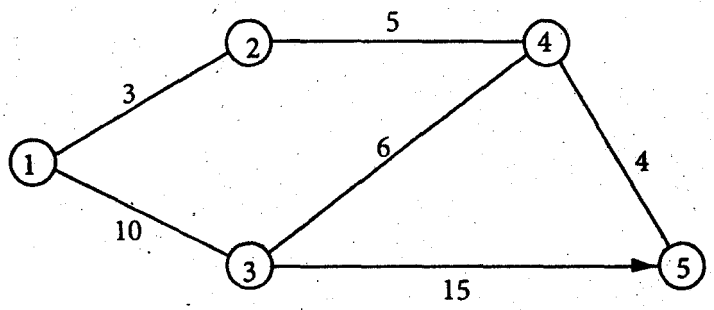
**and** $x_1, x_3 \geq 0$, $x_2$ **Unrestricted in sign**

5. **In a rectangular game the payoff matrix is given below.** **5**

| | | | | |
|---|---|---|---|---|
| 10 | 5 | 5 | 20 | 4 |
| 11 | 15 | 10 | 17 | 25 |
| 7 | 12 | 8 | 9 | 8 |
| 5 | 13 | 9 | 10 | 5 |

State giving reasons, whether the players will use pure or mixed strategies. What is the value of the game?

6. Find the shortest path between every two nodes by Floyd's algorithm.




Hence show the path from node 1 to node 5 and what the shortest distance between them.

## GROUP - C

### ( Long Answer Type Questions )

### Answer any three of the questions

3x15=45

7. a) Using Simplex method solve the following LLP:

Minimize $Z = 4x_1 + 3x_2$

Subject to $x_1 + 2x_2 \geq 8$

$3x_1 + 2x_2 \geq 12$

and $x_1, x_2 \geq 0$ by Charnes Big M method.

b) Find out the initial basic feasible solution and the corresponding transportation cost of the following transportation problem by using VAM:

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | Supply |
|---|---|---|---|---|---|
| $O_1$ | 21 | 16 | 25 | 13 | 110 |
| $O_2$ | 17 | 18 | 14 | 23 | 130 |
| $O_3$ | 32 | 17 | 18 | 41 | 190 |
| Demand | 60 | 100 | 120 | 150 | |

7+8

8. a) Formulate the dual of the primal LPP. Hence find the solution

$$\text{Max } z = x_1 + x_2 + 3x_3$$

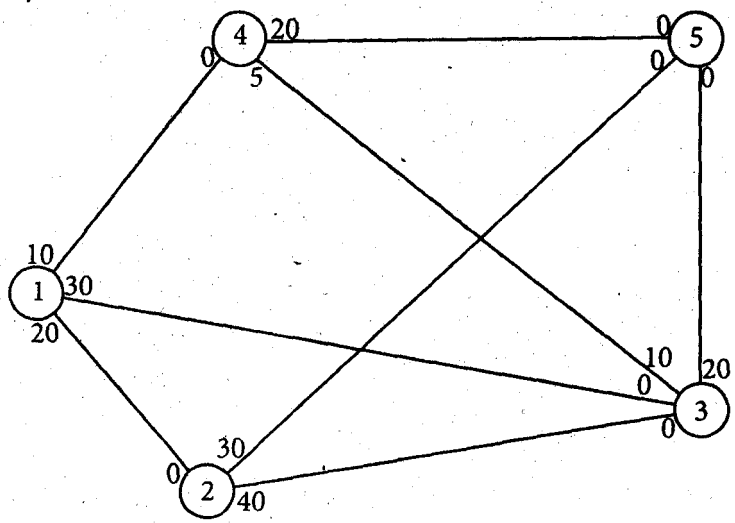
$$\text{S. t. } x_1 + x_2 \leq 8$$

$$x_1 + 2x_2 \leq 5$$

$$2x_1 + x_2 \leq 8$$

$$x_1, x_2 \geq 0$$

b) Find the maximum flow in the network.




9. a) ABC bank has an ATM in a certain locality of Kolkata. The bank authority has received serveral complaints about exceptionally long waiting time in the queue before being served. It is assumed that the service time of a customer by the ATM machine follows exponential distribution. The bank authority has a guideline of keeping the average waiting time of a customer (before being served) within 5 minutes. On receipt of the complaint, the bank authority conducted a survey and found out that the average rate of customer arrival is 15 per hour. If on average the ATM takes 3 minutes to serve a customer then explain whether the administration should open another ATM in that locality. (Assume that the customer arrivals follow a Poisson process)

b) Solve the following game to find out optimal strategies for both the players and the value of the game.

| | | Player B | | | |
|---|---|---|---|---|---|
| | | $B_1$ | $B_2$ | $B_3$ | $B_4$ |
| Player A | $A_1$ | 3 | 2 | 4 | 0 |
| | $A_2$ | 3 | 4 | 2 | 4 |
| | $A_3$ | 4 | 2 | 4 | 0 |
| | $A_4$ | 0 | 4 | 0 | 8 |

8+7

10. a) Solve the Assignment problem :

| | D1 | D2 | D3 | D4 |
|---|---|---|---|---|
| J1 | 8 | 26 | 17 | 11 |
| J2 | 13 | 28 | 4 | 26 |
| J3 | 28 | 19 | 18 | 15 |
| J4 | 19 | 26 | 24 | 10 |

b) Prove that the probability of n customer in a (M/M/1): ($\infty$/FIFO) Queue model is $P_n = \rho^n(1-\rho)$, where $\rho$ is the traffic intensity. Also drive the expected queue length $L_Q = \frac{\rho^2}{1-\rho}$ and the expected length of system is $L_S = \frac{\rho}{1-\rho}$.

8+7

11. a) In a railway marshalling yard, goods trains arrival time follows an exponential distribution and the service time (the time taken to load a train in the hump yard) distribution is also exponential with an average 36 minute. Calculate the following:

(i) the average number of trains in the queue

(ii) the probability that the queue size exceeds 10

(iii) expected waiting time in the queue.

b) Neon lights in a university campus are replaced at a rate of 100 units per day. The maintenance department orders the neon lights periodically. It costs Rs 100 to initiate a purchase order. A neon light kept in storage is estimated to cost about Rs 0,02 per day. The lead time between placing and receiving an order is 12 day. Determine the optimal inventory policy for ordering the neon lights.

8+7

---