MAULANA ABUL KALAM AZAD UNIVERSITY OF TECHNOLOGY, WEST BENGAL

Paper Code : CS-503

DISCRETE MATHEMATICS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) If we divide – 10 with 6 then the remainder will be

a) – 4

b) 4

c) 2

d) – 2.

ii) In the set $S = \{ 1, 2, 3, 4, 6, 9 \}$ defines a relation R by $_a R_b$ if and only if b is a multiple of a. Then which one of the following statements is correct ?

a) 3 and 4 are comparable

b) 9 succeeds 3

c) 3 succeeds 9

d) 4 and 6 are comparable.

iii) The dual of the statement

$(a \wedge b)$ is $\vee\, a = a \wedge (b \vee a)$ is

a) $(a \vee b) \wedge a = a \wedge (b \vee a)$

b) $(a \vee b) \wedge a = a \vee (b \wedge a)$

c) $(a \wedge b) \wedge a = a \vee (b \vee a)$

d) $(a \wedge b) \vee a = a \vee (b \wedge a)$.

iv) The total number of positive divisors of 9216 is

a) 33 b) 20

c) 12 d) 14.

v) A non-empty finite poset has

a) at most one greatest element

b) at most one least element

c) either (a) or (b)

d) both (a) and (b).

vi) The solution of the recurrence relation

$S_n = 2S_{n-1}$ with $S_o = 1$ is $S_n =$

a) 2^n b) 2^{n-1}

c) 2^{n+1} d) none of these.

vii) If for a graph G, $\chi(G) = 10$ and $P(G, \lambda)$ represents chromatic polynomial of G the $P(G, \lambda) = 0$ for

a) $\lambda < 10$ b) $\lambda > 10$

c) $\lambda = 10$ d) none of these.

viii) If $68 \equiv 4 \pmod{n}$, then n can be

a) 12 b) 17

c) 13 d) 16.

ix) $P \rightarrow (P \vee Q)$ is a

a) tautology b) contradiction

c) contingency d) none of these.

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x) Total number of functions from a set of 10 elements of another set of 15 elements is

a) 10^{15} b) 15^{10}

c) 2^{15} d) 2^{10}.

xi) A DNF of $P \rightarrow Q$ is a

a) $\neg P \vee Q$

b) $P \vee \neg Q$

c) $(\neg P \wedge Q) \vee (P \vee \neg Q)$

d) none of these.

xii) The generating function for the sequence

$\{0, 0, 1, 1, 1, \ldots\ldots\ldots\}$ is

a) $\frac{1}{1-x}$

b) $\frac{x^2}{1-x}$

c) $\frac{x^2}{(1-x)^2}$

d) none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Show that $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$ is a tautology.

3. Find the gcd (595, 252) and express it in the form 252m + 595n.

4. Prove that in a distributive lattice if complement of an element exists then it is unique.

5. Find the remainder when the sum

   $1! + 2! + 3! + \ldots\ldots + 100!$ is divided by 18.

6. Prove that the chromatic number of a complete graph with n vertices (K_n) is n.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) A new flag is to be designed with 6 vertical stripes using 4 colours. In how many ways can this be done so that no two adjacent stripes have the same colour ? 5

   b) Find CNF of $\neg (p \vee q) \leftrightarrow (p \wedge q)$ using laws of proposition. 5

   c) Define CRS (mod m). (Complete residue system modulo m). Find all CRS (mod 5). 1 + 4

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8. a) Solve the recurrence relation :

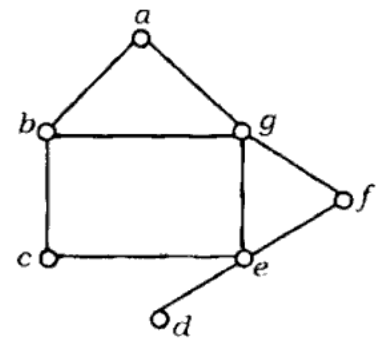
   $$a_{n+2} - 5a_{n+1} + 6a_n = n^2 .$$ 5

   b) Show that t is a valid conclusion from premises $p \rightarrow q$, $q \rightarrow r$, $r \rightarrow s$, $\sim s$ and $p \vee t$. 5

   c) Consider K_6, the complete graph on the six vertices a, b, c, d, e and f. The graph G_1 is obtained from K_6 by deleting the edge ab. The graph G_2 is obtained from G_1 by deleting the edge cd. What are the chromatic numbers of G_1 and G_2 ? 5

9. a) Find integer x and y such that $512x + 320y = 64$. 5

   b) Find maximal matchings, maximum matchings and matching number of the following graph :



   Find if there exists any perfect matching in the graph. 2 + 1 + 1 + 1

   c) Draw the Hasse diagram for the partially ordered set $S = \{2, 3, 5, 30, 60, 120, 180, 360\}$ under the relation division. Hence find maximal, minimal, greatest and least element, if exists. 5

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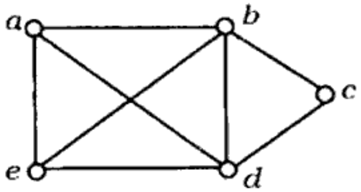
10. a) Find the chromatic polynomial of the following graph G :



b) State Hall's marriage theorem with a statement of marriage condition. There are four processors p_1, p_2, p_3 and p_4 which are designated for six tasks t_1, t_2, t_3, t_4, t_5 and t_6 as follows :

$p_1 \rightarrow \{t_1, t_2\}$, $p_2 \rightarrow \{t_1, t_3, t_4\}$, $p_3 \rightarrow \{t_3, t_4, t_5\}$ and $p_4 \rightarrow \{t_2, t_6\}$. Is it possible to develop a super processor which will serve exactly one task assigned to each of the four processors ?

c) Let C_n be a cycle with n vertices. If C_n is a subgraph of a graph G and n is odd, then show that chromatic number of G *i.e.* $\chi(G) \geq 3$.

11. a) Let m, n be integers not both zero. Prove that $gcd(km, kn) = k.gcd(m, n)$ for any positive integer k.

b) Show that every complete graph is perfect.

c) Let D be a square drawn in the plane with sides of length $\sqrt{2}$. Prove that in every set of 5 distinct points in D, there exists two points whose distance from one another is at most 1.