MCS-301

NUMERICAL METHODS

Time Allotted: 3 Hours | Full Marks: 70

The questions are of equal value.
The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

GROUP A
(Multiple Choice Type Questions)

1. Answer any *ten* questions. $10 \times 1 = 10$

(i) If E_a is the absolute error in a quantity whose true and approximate value are given by x_t and x_a, then the relative error is given by

(A) $\left|\frac{E_a}{x_a}\right|$ (B) $\left|\frac{E_a}{x_t}\right|$

(C) $\left|\frac{E_a}{x_t - x_a}\right|$ (D) $|E_a|$

(ii) Fixed point iteration method is

(A) conditionally convergent (B) divergent

(C) linearly convergent (D) none of these

(iii) Which of the following is not true (the notation have their usual meaning)?

(A) $\Delta = E - I$ (B) $\Delta \cdot \nabla = \Delta - \nabla$

(C) $\frac{\Delta}{\nabla} = \Delta + \nabla$ (D) $\nabla = I - E^{-1}$

(iv) The degree of precision of Simpson's $\frac{1}{3}$rd rule is

(A) 1 (B) 2

(C) 3 (D) 4

(v) Condition of convergence of Newton-Raphson method is

(A) $|f(x).f'(x)| < \{f''(x)\}^2$ (B) $|f(x).f''(x)| < \{f'(x)\}^2$

(C) $|f(x).f'(x)| > \{f''(x)\}^2$ (D) $|f(x).f''(x)| > \{f'(x)\}^2$

(vi) In Newton's backward interpolation, the interval should be

(A) equally spaced (B) not equally spaced

(C) may be equally spaced (D) both (A) and (B)

(vii) The percentage error in approximation 1/3 to 0.3333 is

(A) 0.06% (B) 0.006%

(C) 0.6% (D) 6%

(viii) Runge-Kutta method is used to solve

(A) an algebraic equation

(B) a first order ordinary differential equation

(C) a first order partial differential equation

(D) none of these

(ix) Rounding off the number 0.03709157 correct upto 5 significant figure is

(A) 0.03709 (B) 0.037091

(C) 0.037092 (D) 0.0370

(x) The truncation error of Euler's method is

(A) $O(h)$ (B) $O(h^3)$

(C) $O(h^2)$ (D) $O(h^4)$

(xi) Gauss elimination method does not fail even if one of the pivotal elements is equal to zero:

(A) true (B) false

(C) neither true nor false (D) none of these

(xii) If the interval of differencing in unity and $f(x) = ax^2$ (a is constant), which one of the following choices is wrong?

(A) $\Delta f(x) = a(2x + 1)$ (B) $\Delta^2 f(x) = 2a$

(C) $\Delta^3 f(x) = 2$ (D) $\Delta^4 f(x) = 0$

GROUP B
(Short Answer Type Questions)

3×5 = 15

Answer any *three* questions.

2. Use Lagrange's Interpolation formula to find the value of $y = f(x)$ for $x = 1$, given in the following table:

x	0	2	3	3
y	0	8	15	35

3

3. (a) Prove that $\Delta \cdot \nabla = \Delta - \nabla$. 2

(b) Evaluate $\Delta^2 \tan 2x$.

4. Solve by using Modified Euler method the following differential equation for $x = 1$ by taking $h = 0.1$: $\frac{dy}{dx} = x + y$, $y = 1$ when $x = 0$.

5. Evaluate $\int \frac{x}{\sin x} dx$, where the interval is (0, 2) by using Trapezoidal rule taking $n = 8$.

6. Find inverse of the following matrix by Gauss-Jacobi method $\begin{pmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{pmatrix}$.

GROUP C
(Long Answer Type Questions)

Answer any *three* questions. 3×15 = 45

7. (a) $\Delta^n\left(\frac{1}{x}\right) = \frac{(-1)^n n! h^n}{x(x+h)(x+2h)...(x+nh)}$ 5

(b) Deduce Simpson's $\frac{1}{3}$ *rd rule* (from Newton-Cote's quadrature formula). 6

(c) What is the geometrical interpretation of Trapezoidal rule? 4

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8. (a) Find the root of the equation $x - 3x - 4 = 0$, that lies between 1 and 2, correct upto 3 decimal places using the Bisection Method. State the advantages and disadvantages of this method. 6+2

(b) What is the lowest degree polynomial which takes the following values: 5+2

x	0	2	3	4	7	8
$f(x)$	4	26	58	112	466	668

Hence find $f(5)$.

9. (a) Solve the equation $\frac{dy}{dx} = \frac{1}{xy}$, $y = 1$ when $x = 0$, for $y(0.1)$, $y(0.2)$ and $y(0.3)$ using Runge-Kutta method of the fourth order. 6

(b) Solve the system of linear equations by LU-factorisation method: 5

$$2x - 6y + 8z = 24$$
$$5x + 4y - 3z = 2$$
$$3x + y + 2z = 16$$

(c) Find the fourth degree curve $y = f(x)$ passing through the points (2, 3), (4, 43), (7,778) and (8, 1515) using Newton's divided difference formula. 4

10. (a) The following data represents the function $f(x) = e^x$. 10

x	1	1.5	2	2.5
y	2.7183	4.4817	7.3891	12.1825

Estimate the value of $f(1.25)$ using (i) Newton's forward difference interpolation (ii) Newton's backward difference interpolation.

(b) Prove the given relation : $\delta = \nabla \cdot E^{0.5}$ 5

11. (a) Solve by Gauss elimination method: 6

$$x + 2y + z = 0$$
$$2x + 2y + 3z = 3$$
$$-x - 3y = 2$$

(b) Find the root of the equation $3x^3 - 11x^2 + 7x + 2 = 0$, correct to 4 decimal places, using Newton-Raphson method. 6

(c) Explain the difference between Round-Off and Chopping.