



Name : .....

Roll No. : .....

Invigilator's Signature : .....

**CS/B.TECH/FT (O)/CHE (O)/SEM-3/M-315/2011-12**

**2011  
MATHEMATICS**

Time Allotted : 3 Hours

Full Marks : 70

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words  
as far as practicable.*

**GROUP – A  
( Multiple Choice Type Questions )**

1. Choose the correct alternatives for the following :

$$10 \times 1 = 10$$

i) The probability that a leap year selected at random will contain 53 Wednesdays is

a)  $\frac{3}{4}$

b)  $\frac{2}{7}$

c)  $\frac{1}{3}$

d)  $\frac{4}{9}$ .

ii) The Variance of a random variable  $X$  is

a)  $[E(X)]^2$

b)  $E(X^2)$

c)  $E(X^2) - [E(X)]^2$

d)  $[E(X^2)]^2 - [E(X)]$ .

iii) The period of the function  $f(x) = 2|\cos^2 x|$  is

a)  $\pi$

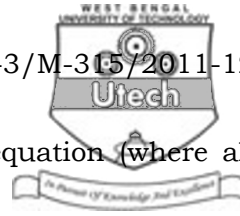
b)  $2\pi$

c)  $\frac{2\pi}{3}$

d)  $\frac{\pi}{3}$ .



- iv) The function  $f(x) = k \sin x$ ,  $0 \leq x < \pi$  is
- a) Half wave Rectifier      b) Full wave rectifier
- c) Triangular waveform      d) none of these.
- v)  $b_{yx} \times b_{xy}$  (where  $b_{yx}$ ,  $b_{xy}$  and  $r$  are regression and correlation coefficients) is
- a)  $r$       b)  $r^2$
- c)  $\frac{1}{r}$       d) none of these.
- vi) The order and degree of  $\sqrt{\frac{\partial z}{\partial x}} + \sqrt{\frac{\partial z}{\partial y}} = x - y$
- (where  $z = f(x, y)$ ) is
- a) 1, 4      b) 1, 2
- c) 1, 3      d) 1, 1.
- vii) The Lagrange's differential equation for linear first order partial derivative (where all the symbols have usual meanings) is
- a)  $Pp + Qq = R$       b)  $P^2 Q^2 = R$
- c)  $Pq + Qp = R$       d)  $Pq - Qp = R$ .
- viii) The order of Bessel's differential equation is
- a) 1      b) 2
- c)  $n$       d) 4.



ix) The general solution of Legendre's equation (where all the symbols have usual meanings) is

- a)  $AP_n(x) + BQ_n(x)$       b)  $P_n(x) Q_n(x)$   
 c)  $\frac{P_n(x)}{Q_n(x)}$       d)  $AP_n(x) - BQ_n(x)$ .

x) The mean and standard deviation of a binomial distribution are respectively 4 and  $\sqrt{\frac{8}{3}}$ . The values of  $n$  and  $p$  are

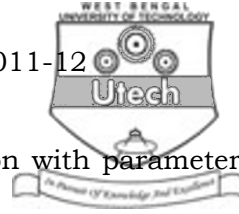
- a)  $11, \frac{3}{4}$       b)  $12, \frac{2}{7}$   
 c)  $12, \frac{1}{3}$       d)  $13, \frac{3}{8}$ .

### GROUP – B

#### ( Short Answer Type Questions )

Answer any *three* of the following.  $3 \times 5 = 15$

2. Solve  $(D^2 - 2DD') z = e^{2x} + x^2 y$ , where  $\left( D = \frac{\partial}{\partial x}, D' = \frac{\partial}{\partial y} \right)$ .
3. Solve  $\sqrt{p} + \sqrt{q} = 1$ , where  $\left( P = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y} \right)$ .



4. Find the mean of the Binomial distribution with parameters  $n, p$ .
5. Let  $x$  and  $y$  be two variables whose means are  $\bar{x}$  and  $\bar{y}$ ; standard deviations are  $\sigma_x$  and  $\sigma_y$  respectively. If  $u = \frac{x - \bar{x}}{\sigma_x}$  and  $v = \frac{y - \bar{y}}{\sigma_y}$ , then show that  $r_{xy} = \text{Cov}(u, v)$ .
6. Find the Fourier series of the function  $f(x) = x - x^2$ ,  $-\pi < x \leq \pi$ .
7. Show that when  $n$  is a positive integer,  $J_{-n}(x) = (-1)^n J_n(x)$ .
8. The 5 pair of values of  $x$  and  $y$  are such that  $\text{Var}(x) = 6$ ,  $\text{Var}(y) = 2$  and  $r_{xy} = 0.98$  (where symbols have their usual meanings). Find  $\text{Var}(2x + 3y)$ .

### GROUP – C

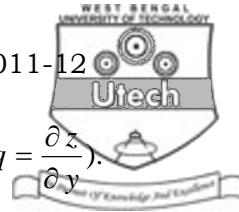
#### ( Long Answer Type Questions )

Answer any *three* of the following.  $3 \times 15 = 45$

9. a) If  $x = 4y + 5$  and  $y = kx + 4$  be two regression equations of  $x$  on  $y$  and  $y$  on  $x$  respectively, then find the interval in which  $k$  lies.
- b) Prove that  $-1 \leq r_{xy} \leq 1$  (where  $r_{xy}$  is the correlation coefficient between two variables  $x$  and  $y$ ).



- c) The bivariate  $(U, V)$  is related with the bivariate  $(X, Y)$  by the two relations  $4U = 2X + 7$  and  $6V = 2Y - 15$ . Given a regression coefficient of  $Y$  on  $X$  is 3. Find the regression coefficient of  $V$  on  $U$ .
10. a) Show that a function,
- $$f(x) = |x|, \quad -1 < x < 1$$
- $$= 0, \quad \text{elsewhere}$$
- is a possible probability density function and hence find the corresponding distribution function.
- b) A radioactive source emits an average 2.5 particles per second. Calculate the probability that 2 or more particles will be emitted in an interval of 4 seconds.
- c) If a person gets Rs  $(2x + 5)$  where  $x$  denotes the number appearing when a balanced die is rolled once, then how much money can be expected in the long run per game?
11. a) Solve the wave equation  $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ , given that  $u(0, t) = u(l, t) = 0$ ,  $u(x, 0) = f(x)$  and  $\frac{\partial u}{\partial t}(x, 0) = 0$ ,  $0 < x < l$
- b) Solve  $(p + q)(z - xp - yq) = 1$ , where  $p = \frac{\partial z}{\partial x}$ ,  $q = \frac{\partial z}{\partial y}$ .



12. a) Solve  $p^2 x + qy = z$ , where  $(p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y})$ .
- b) Find the integral surface satisfying partial differential equation  $(x - y)p + (y - x - z)q = z$  and passing through the circle  $x^2 + y^2 = 1, z = 1$ .
- c) Find the general solution of the equation :  
 $z(p - q) = z^2 + (x + y)^2$  where  $(p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y})$ .
13. a) Find the Fourier series of the function  $e^{-x}$  in the interval  $0 < x < 2\pi$ .
- b) Find the sine series which represents the function  $f(x) = \pi - x$  in  $0 < x < \pi$ .
- c) Write Parseval's identity corresponding to Half range cosine series of the function  $f(x) = x, 0 < x < 2$ .  
 Hence determine the sum of the series  
 $\frac{1}{1^4} + \frac{1}{2^4} + \dots + \frac{1}{n^4} + \dots$
14. a) Find the power series solution of the equation  
 $(1 - x^2) \frac{d^2 y}{dx^2} + 2y = 0$ , given that  $y(0) = 4, y'(0) = 5$ .
- b) Find the general solution of the differential equation  
 $(1 - x^2) y'' - 2xy' + n(n + 1)y = 0$ .



15. a) Express  $x^4 - 3x^2 + x$  as a series solution in Legendre's polynomial.
- b) Expand  $\cos px$  in  $[-\pi, \pi]$  ( $p$  not being an integer) in Fourier series.

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