Name : ....
Roll No. : ....
Invigilator's Signature : ....

CS/B.TECH/FT (O)/CHE (O)/SEM-3/M-315/2011-12

2011

MATHEMATICS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for the following :

$10 \times 1 = 10$

i) The probability that a leap wear selected at random will contain 53 Wednesdays is

a) $\frac{3}{4}$ b) $\frac{2}{7}$

c) $\frac{1}{3}$ d) $\frac{4}{9}$.

ii) The Variance of a random wariable X is

a) $[E(X)]^2$ b) $E(X^2)$

c) $E(X^2) - [E(X)]^2$ d) $[E(X^2)]^2 - [E(X)]$.

iii) The period of the function $f(x) = 2|\cos^2 x|$ is

a) π b) 2π

c) $\frac{2\pi}{3}$ d) $\frac{\pi}{3}$.

iv) The function $f(x) = k \sin x,\ 0 \le x < \pi$ is

a) Half wave Rectifier b) Full wave rectifier

c) Triangular waveform d) none of these.

v) $b_{yx} \times b_{xy}$ (where b_{yx} , b_{xy} and r are regression and correlation coefficients) is

a) r b) r^2

c) $\frac{1}{r}$ d) none of these.

vi) The order and degree of $\sqrt{\frac{\partial z}{\partial x}} + \sqrt{\frac{\partial z}{\partial y}} = x - y$

(where $z = f(x, y)$) is

a) 1, 4 b) 1, 2

c) 1, 3 d) 1, 1.

vii) The Lagrange's differential equation for linear first order partial derivative (where all the symbols have usual meanings) is

a) $Pp + Qq = R$ b) $P^2 Q^2 = R$

c) $Pq + Qp = R$ d) $Pq - Qp = R$.

viii) The order of Bessel's differential equation is

a) 1 b) 2

c) n d) 4.

ix) The general solution of Legendre's equation (where all the symbols have usual meanings) is

a) $AP_n(x) + BQ_n(x)$
b) $P_n(x)\ Q_n(x)$
c) $\dfrac{P_n(x)}{Q_n(x)}$
d) $AP_n(x) - BQ_n(x)$.

x) The mean and standard deviation of a binomial distribution are respectively 4 and $\sqrt{\dfrac{8}{3}}$. The values of n and p are

a) 11, $\dfrac{3}{4}$
b) 12, $\dfrac{2}{7}$
c) 12, $\dfrac{1}{3}$
d) 13, $\dfrac{3}{8}$.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Solve $(D^2 - 2DD')\,z = e^{2x} + x^2 y$, where $\left(D = \dfrac{\partial}{\partial x}, D' = \dfrac{\partial}{\partial xy} \right)$.

3. Solve $\sqrt{p} + \sqrt{q} = 1$, where $\left(P = \dfrac{\partial z}{\partial x}, q = \dfrac{\partial z}{\partial y} \right)$.

4. Find the mean of the Binomial distribution with parameters n, p.

5. Let x and y be two variables whose means are $\bar{x}$ and $\bar{y}$; standard deviations are σ_x and σ_y respectively. If $u = \dfrac{x-\bar{x}}{\sigma_x}$ and $v = \dfrac{y-\bar{y}}{\sigma_y}$, then show that $r_{xy} = \text{Cov}(u, v)$.

6. Find the Fourier series of the function $f(x) = x - x^2$, $-\pi < x \leq \pi$.

7. Show that when n is a positive integer, $J_{-n}(x) = (-1)^n J_n(x)$.

8. The 5 pair of values of x and y are such that $Var(x) = 6$, $Var(y) = 2$ and $r_{xy} = 0.98$ (where symbols have their usual meanings). Find $Var(2x + 3y)$.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

9. a) If $x = 4y + 5$ and $y = kx + 4$ be two regression equations of x on y and y on x respectively, then find the interval in which k lies.

   b) Prove that $-1 \leq r_{xy} \leq 1$ (where r_{xy} is the correlation coefficient between two variables x and y).

c) The bivariate (U, V) is related with the bivariate (X, Y) by the two relations $4U = 2X + 7$ and $6V = 2Y - 15$. Given a regression coefficient of Y on X is 3. Find the regression coefficient of V on U.

10. a) Show that a function,

$$f(x) = |x| \quad , \quad -1 < x < 1$$
$$= 0 \qquad , \quad \text{elsewhere}$$

is a possible probability density function and hence find the corresponding distribution function.

b) A radioactive source emits an average 2.5 particles per second. Calculate the probability that 2 or more particles will be emitted in an interval of 4 seconds.

c) If a person gets Rs $(2x + 5)$ where x denotes the number appearing when a balanced die is rolled once, then how much money can be expect in the long run per game ?

11. a) Solve the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$, given that $u(0, t) = u(l, t) = 0$, $u(x, 0) = f(x)$ and $\frac{\partial u}{\partial t}(x, 0) = 0$, $0 < x < l$

b) Solve $(p + q)(z - xp - yq) = 1$, where $\left(p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}\right)$.

12. a) Solve $p^2 x + qy = z$, where ($p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$).

b) Find the integral surface satisfying partial differential equation $(x - y)\, p + (y - x - z)\, q = z$ and passing through the circle $x^2 + y^2 = 1$, $z = 1$.

c) Find the general solution of the equation :

$$z(p - q) = z^2 + (x + y)^2 \text{ where } \left(p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y} \right).$$

13. a) Find the Fourier series of the function e^{-x} in the interval $0 < x < 2\pi$.

b) Find the sine series which represents the function $f(x) = \pi - x$ in $0 < x < \pi$.

c) Write Parseval's identity corresponding to Half range cosine series of the function $f(x) = x$, $0 < x < 2$. Hence determine the sum of the series $\frac{1}{1^4} + \frac{1}{2^4} + \ldots\ldots + \frac{1}{n^4} + \ldots\ldots$

14. a) Find the power series solution of the equation

$$(1 - x^2)\frac{d^2 y}{dx^2} + 2y = 0, \text{ given that } y(0) = 4,\ y'(0) = 5.$$

b) Find the general solution of the differential equation $(1 - x^2)\, y'' - 2xy' + n(n+1)\, y = 0$.

15. a) Express $x^4 - 3x^2 + x$ as a series solution in Legendre's polynomial.

b) Expand $\cos px$ in $[-\pi, \pi]$ (p not being an integer) in Fourier series.