

Name :
Roll No. :
Invigilator's Signature :

CS/B.Tech (NEW)/SEM-1/M-101/2011-12
2011
MATHEMATICS - I

Time Allotted : 3 Hours Full Marks : 70

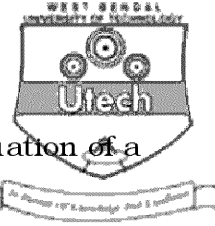
The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words
as far as practicable.

GROUP - A
(Multiple choice Type Questions)

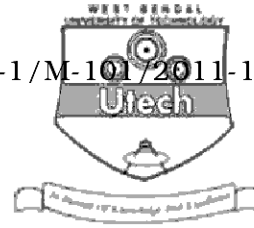
1. Choose the correct alternatives for any *ten* of the following :
 $10 \times 1 = 10$

- i) The least upper bound of the sequence $\{\frac{n}{n+1}\}$ is
- a) 0
 - b) $\frac{1}{2}$
 - c) 1
 - d) 2.

- ii) The value of $\begin{vmatrix} 2000 & 2001 & 2002 \\ 2003 & 2004 & 2005 \\ 2006 & 2007 & 2008 \end{vmatrix}$ is
- a) 2000
 - b) 0
 - c) 45
 - d) none of these.



- iii) If $\lambda^3 - 6\lambda^2 + 9\lambda - 4$ is the characteristic equation of a square matrix A, then A^{-1} is equal to
- a) $A^2 - 6A + 9I$ b) $\frac{1}{4}A^2 - \frac{3}{2}A + \frac{9}{4}I$
- c) $A^2 - 6A + 9$ d) $\frac{1}{4}A^2 - \frac{3}{2}A + \frac{9}{4}$
- iv) If $x = r \cos \theta$, $y = r \sin \theta$, then $\frac{\partial(r, \theta)}{\partial(x, y)}$ is
- a) r b) 1
- c) $\frac{1}{r}$ d) none of these.
- v) $f(x, y) = \frac{\sqrt{y} + \sqrt{x}}{y + x}$ is a homogeneous function of degree
- a) $\frac{1}{2}$ b) $-\frac{1}{2}$
- c) 1 d) 2 .
- vi) If $\vec{\alpha} \cdot (\vec{\beta} \times \vec{\gamma}) = 0$, then the vectors $\vec{\alpha}$, $\vec{\beta}$, $\vec{\gamma}$ are
- a) coplanar b) independent
- c) collinear d) none of these.
- vii) The n th derivative of $(ax+b)^{10}$ is (where $n > 10$)
- a) a^{10} b) $\underline{10} a^{10}$
- c) 0 d) $\underline{10}$.
- viii) If for any two vectors \vec{a} and \vec{b} , $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, then \vec{a} and \vec{b} are
- a) parallel b) collinear
- c) perpendicular d) none of these.



ix) If $A^{-1} = \frac{1}{7} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, then $A =$

a) $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ b) $\begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$

c) $\frac{1}{7} \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$ d) $\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$.

x) The reduction formula of $I_n = \int^{\frac{\pi}{2}} \cos^n x dx$ is

a) $I_n = \left(\frac{n-1}{n}\right) I_{n-1}$ b) $I_n = \left(\frac{n}{n-1}\right) I_n$

c) $I_n = \left(\frac{n-1}{n}\right) I_{n-2}$ d) none of these.

xi) The series $\sum_{n=1}^{\infty} \frac{n^2}{2n^2 + 1}$ is

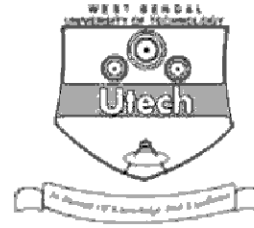
a) convergent b) divergent

c) oscillatory d) none of these.

xii) Lagrange's Mean Value Theorem is obtained from Cauchy's Mean Theorem for *two* functions $f(x)$ and $g(x)$ by putting $g(x) =$

a) 1 b) x^2

c) x d) $\frac{1}{x}$.



GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Prove that every square matrix can be expressed as the sum of a symmetric matrix and a skew symmetric matrix :

3. By Laplace's method, prove that

$$\begin{vmatrix} a & b & c & d \\ -b & a & d & -c \\ -c & -d & a & b \\ -d & c & -b & a \end{vmatrix} = (a^2 + b^2 + c^2 + d^2)^2$$

(consider minors of order 2).

4. If $2x = y^m + y^{-m}$, then prove that

$$(x^2 - 1)y_{n+2} + (2n + 1)xy_{n+1} + (n^2 - m^2)y_n = 0$$

5. If $u = xf\left(\frac{y}{x}\right) + g\left(\frac{y}{x}\right)$, then show that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 xy}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 0$$

6. Show that the bounded by a simple closed curve C is given

$$\text{by } \frac{1}{2} \oint_C (xdy - ydx)$$



GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. i) If $f(x,y) = x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right)$, verify $f_{xy} = f_{yx}$.
- ii) State the Rolle's theorem and examine if you can apply the same for $f(x) = \tan x$ in $[0, \pi]$
- iii) Find the value of λ and μ for which

$$x + y + z = 3$$

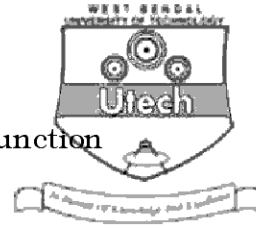
$$2x - y + 3z = 4$$

$$5x - y + \lambda z = \mu \quad \text{have}$$

(a) unique solution

(b) many solution

(c) no solution.



8. i) Find the maxima and minima of the function

$$f(x,y) = x^3 + y^3 - 63(x+y) + 12xy.$$

Find also the saddle points.

- ii) State Leibnitz's test for alternating series and apply it to examine the convergence of $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots \alpha$.

- iii) Applying Lagrange's Mean Value Theorem prove that

$$\frac{x}{1+x} \leq \log(1+x) < x, \text{ for all } x > 0.$$

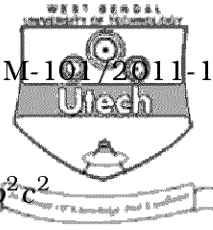
9. i) If $y = e^{m \sin^{-1} x}$, show that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0 \text{ Hence find } y_n$$

when $x = 0$.

- ii) Prove that $[a+b \ b+c \ c+a] = 2[a \ b \ c]$, where a, b, c are three vectors.

- iii) Find the directional derivative of $f = xyz$ at $(1,1,1)$ in the direction $2\vec{i} - \vec{j} - 2\vec{k}$.



10. i) Prove that
$$\begin{vmatrix} b^2+c^2 & a^2 & a^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix} = 4a^2b^2c^2$$

- ii) State Divergence Theorem of Gauss. Verify divergence theorem for $\vec{F} = y\vec{i} + x\vec{j} + z\vec{k}$ over the cylindrical region bounded by $x^2 + y^2 = 9, z = 0, z = 2$.
- iii) Test the series for convergence

$$\frac{1^p}{2^q} + \frac{2^p}{3^q} + \frac{3^p}{4^q} + \dots$$

11. i) Obtain a reduction formula for $\int_0^{\frac{1}{2}} \sin^n x dx$. Hence obtain

$$\int_0^{\frac{\pi}{2}} \sin^9 x dx$$

- ii) Given two vectors $\vec{\alpha} = 3\vec{i} - \vec{j}$, $\vec{\beta} = 2\vec{i} + \vec{j} - 3\vec{k}$. Express $\vec{\beta}$ in the form $\vec{\beta}_1 + \vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$.
- iii) Show that $\vec{A} = (6xy + z^3)\vec{i} + (3x^2 - z)\vec{j} + (3xz^2 - y)\vec{k}$ is irrotational. Find the scalar function ϕ , such that $\vec{A} = \nabla\phi$.